Topology optimization of a porous material by homogenization for the conception of an air-oil separator

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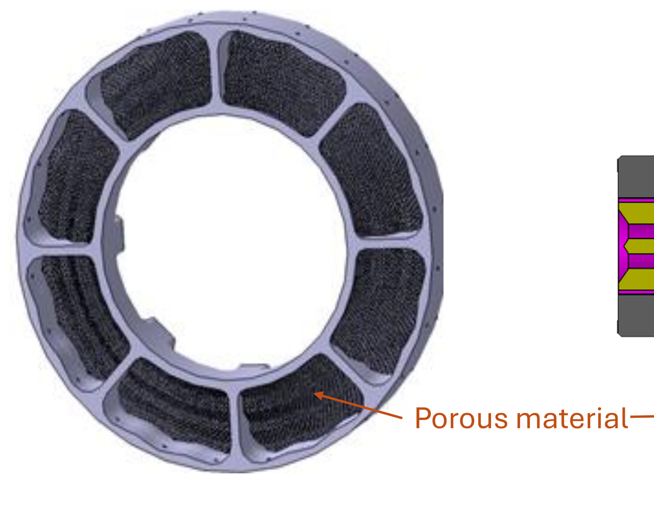
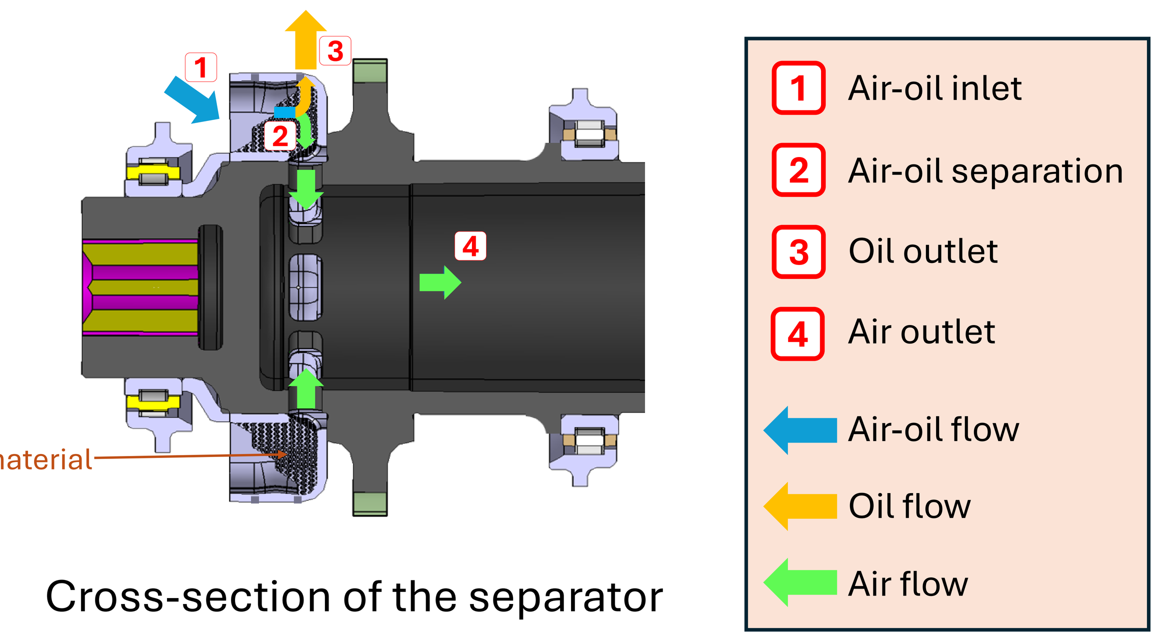
Motivation/Context

- Aircraft engines are fitted with an **air-oil separator**[1] designed to filter out oil droplets from the air stream while allowing as much air as possible to pass through, using a porous material.
- The oil is collected along the outer borders of the separator by centrifugal force while the air passes through the porous medium and is released into the atmosphere.
- The shape of the porous material can be optimized to minimize oil consumption while limiting pressure drop.



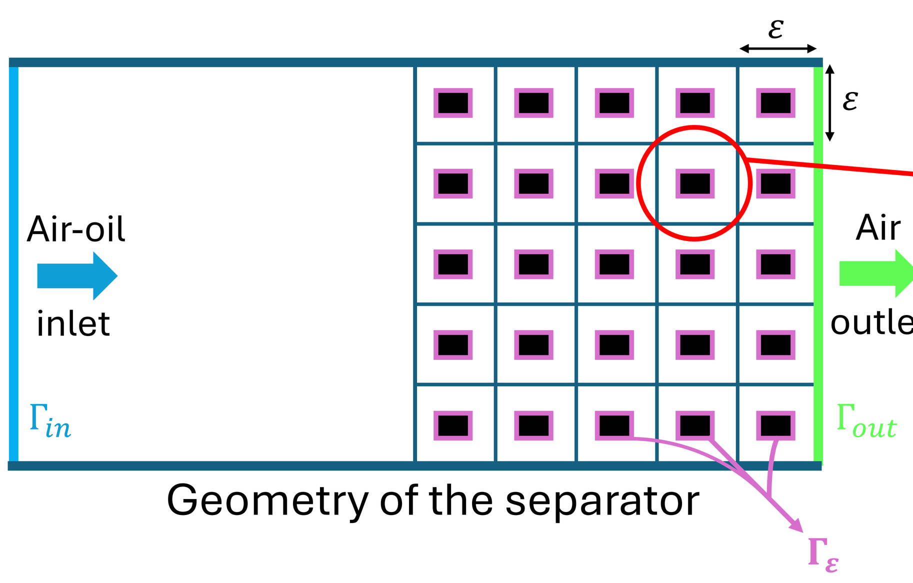
Air-oil separator



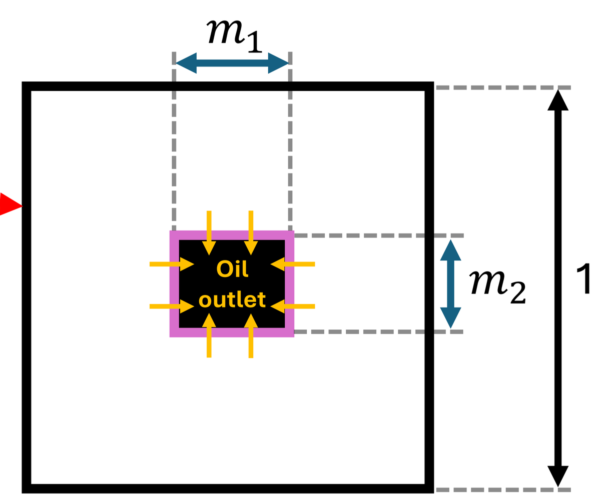
Cross-section of the separator

Model[2]

- **Porous material**
  - Domain $\Omega^\varepsilon = \Omega_1 \cup \Omega_2^\varepsilon$ where Ω_2^ε is divided into periodic cells of size ε
  - Microstructure : a square that contains a rectangular obstacle with side lengths (m_1, m_2)
- **Air-oil fluid**
  - A fluid consisting of air and oil, assumed to be **single-phase**
  - Incompressible **Navier-Stokes** equations with **Robin** boundary conditions at obstacle boundaries Γ_ε representing the oil droplet capture



Geometry of the separator



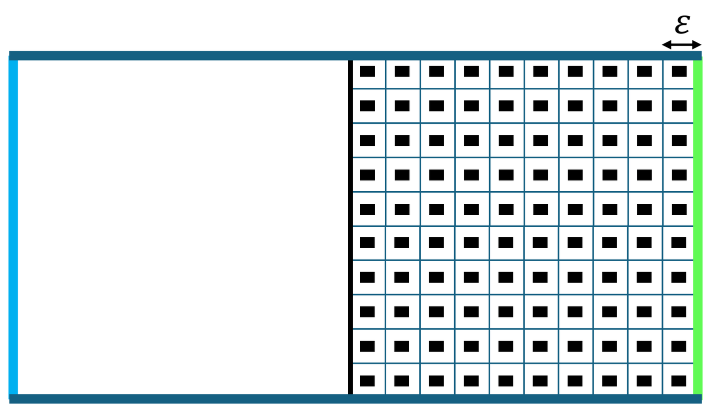
Rescaled periodic cell

Homogenization [3 4]

- **Homogenization**
  - Average of physical properties of the porous material
  - Effective material with **no obstacle** representing the asymptotic behaviour of the porous material as ε approaches 0
  - New homogenized equations in fixed domain $\Omega = \Omega_1 \cup \Omega_2$
  - **No incompressibility condition** on porous medium Ω_2
- **Optimization parameter**
  - Initial model : $\{\boldsymbol{m^i} = (m_1^i, m_2^i) \in [0,1]^2, i \in \{1, \dots, \#obstacles(\varepsilon)\}\}$
  - Homogenized model : $\boldsymbol{m} = (m_1, m_2) \in L^\infty(\Omega_2;\ [0,1]^2)$

Initial model

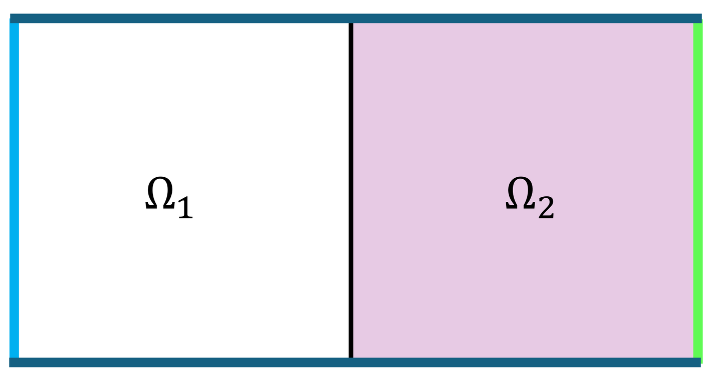
$$\begin{cases} -2\nu \operatorname{div} e(\boldsymbol{u^\varepsilon}) + \boldsymbol{u^\varepsilon}\cdot\nabla\boldsymbol{u^\varepsilon} + \nabla p^\varepsilon = \boldsymbol{0} & \text{in } \Omega^\varepsilon(\boldsymbol{m}) \\ \operatorname{div}(\boldsymbol{u^\varepsilon}) = 0 & \\ \boldsymbol{u^\varepsilon} = \boldsymbol{u_0} & \text{on } \partial\Omega_{ext} \\ (2\nu e(\boldsymbol{u^\varepsilon}) - p^\varepsilon Id)\boldsymbol{n}\cdot\boldsymbol{n} = -\alpha^\varepsilon \boldsymbol{u^\varepsilon}\cdot\boldsymbol{n} & \text{on } \Gamma_\varepsilon(\boldsymbol{m}) \\ (2\nu e(\boldsymbol{u^\varepsilon}) - p^\varepsilon Id)\boldsymbol{n}\cdot\boldsymbol{t} = 0 & \end{cases}$$



$\varepsilon \to 0$ $\qquad$ $\varepsilon \to 0$

Homogenized model

$$\begin{cases} -2\nu \operatorname{div} e(\boldsymbol{u}) + \boldsymbol{u}\cdot\nabla\boldsymbol{u} + \nabla p = \boldsymbol{0} & \text{in } \Omega_1 \\ \operatorname{div}(\boldsymbol{u}) = 0 & \\ \boldsymbol{H}(\boldsymbol{m}; \boldsymbol{u}, p) = \boldsymbol{0} & \text{on } \Omega_2 \\ \boldsymbol{u} = \boldsymbol{u_0} & \text{on } \partial\Omega_{ext} \end{cases}$$



Optimization

- Two performance criteria : **oil consumption** and **pressure drop**

1. **Oil collected by the separator**

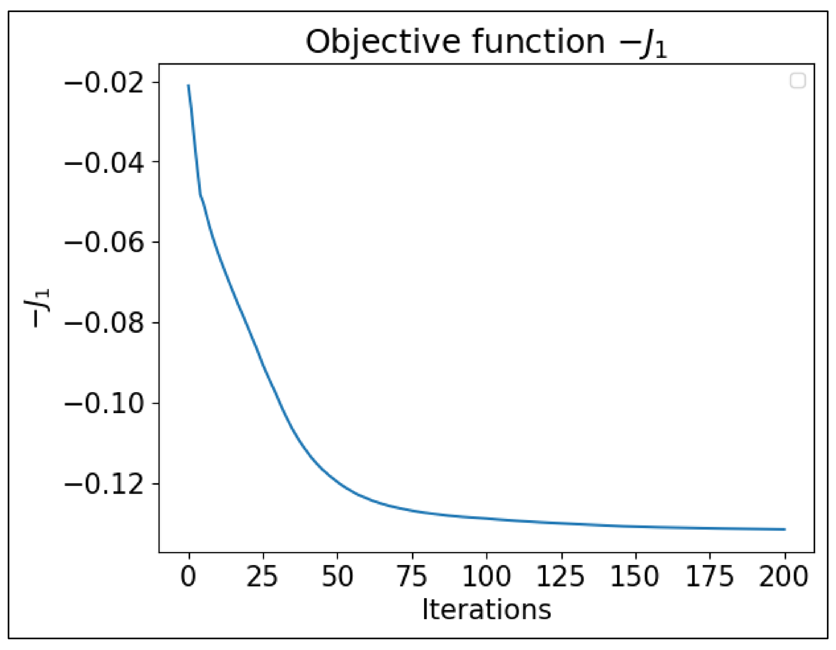
$$J_1^\varepsilon(\boldsymbol{m}) = \int_{\Gamma_\varepsilon} \boldsymbol{u^\varepsilon}\cdot\boldsymbol{n}\, ds \xrightarrow[\varepsilon\to 0]{} J_1(\boldsymbol{m}) = \int_{\Omega_2} \operatorname{div}(\boldsymbol{u})dx$$

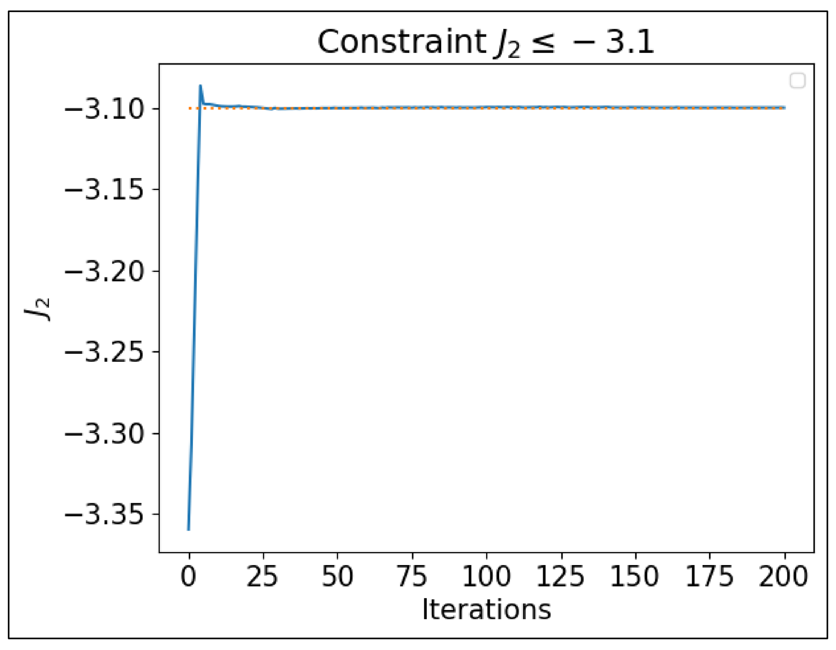
2. **Pressure drop**

$$J_2^\varepsilon(\boldsymbol{m}) = \int_{\Gamma_{out}} (\boldsymbol{p^\varepsilon} - 2\nu\, e(\boldsymbol{u^\varepsilon})\boldsymbol{n}\cdot\boldsymbol{n})\, ds - \int_{\Gamma_{in}} (\boldsymbol{p^\varepsilon} - 2\nu\, e(\boldsymbol{u^\varepsilon})\boldsymbol{n}\cdot\boldsymbol{n})\, ds$$

- Optimization with **Null Space algorithm**[5]
- The oil collected is maximized under a constraint on the pressure drop

$$\min_{\boldsymbol{m}\in L^\infty(\Omega_2;\,[0,1]^2)} -J_1(\boldsymbol{m}) \quad s.t \quad J_2(\boldsymbol{m}) < C$$



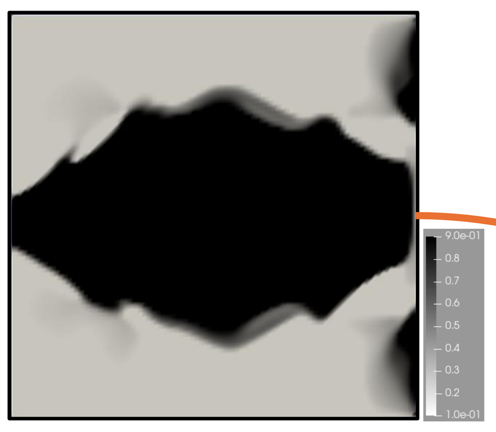


Dehomogenization [6]

- Optimization does not directly yield a manufacturable material
- **Dehomogenization**
  - Interpretation of optimized parameters into an actual material
  - Projection of fields (m_1, m_2) onto a grid of size ε as in the initial model
  - Characterization of final structure with level-set function $F^\varepsilon(\boldsymbol{m}; x)$

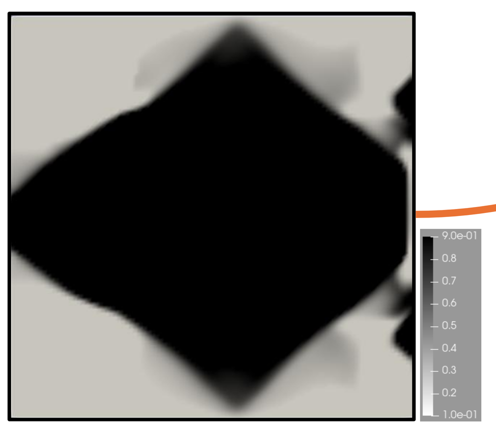
$$\Omega_2^\varepsilon = \{x \in \Omega_2 \text{ such that } F^\varepsilon(\boldsymbol{m}; x) \leq 0\}$$

where
$$\begin{cases} f_i^\varepsilon(\boldsymbol{m}; x) = -\cos\left(\frac{2\pi x_i}{\varepsilon}\right) + \cos(\pi(1 - m_i(x))) \\ F^\varepsilon = \min(f_1^\varepsilon, f_2^\varepsilon) \end{cases}$$

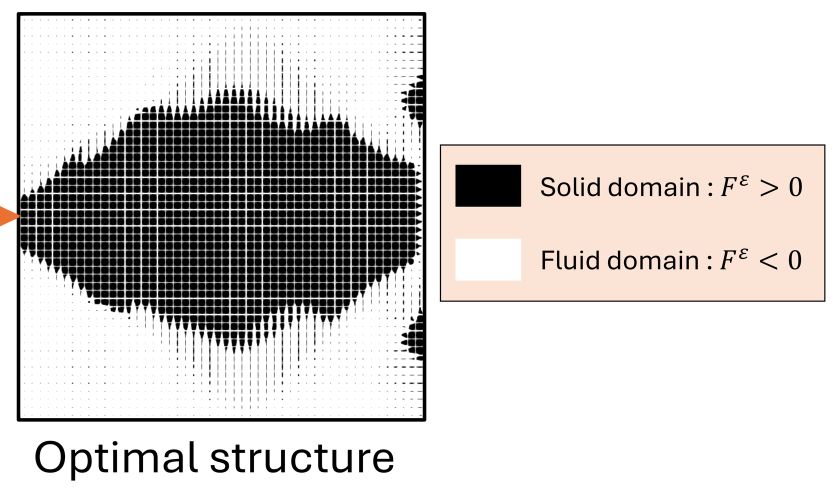


Optimized m_1

Dehomogenization



Optimized m_2



Optimal structure

References

- **[1]** L. Seguinot. Etude et développement d'une stratégie d'analyse des performances d'un dégazeur de turbine d'hélicoptère. Thèse 2017.
- **[2]** C. Conca. Mathematical modeling of the steam-water condensation in a condenser. Large-Scale Computations in Fluid Mechanics, 1985.
- **[3]** G. Allaire. A brief introduction to homogenization and miscellaneous applications. ESAIM : Proceedings, 37 :1–49, 2012.
- **[4]** F. Murat and L. Tartar. Calculus of variations and homogenization. In Topics in the mathematical modelling of composite materials, 1997.
- **[5]** F. Feppon, G. Allaire, and C. Dapogny. Null space gradient flows for constrained optimization with applications to shape optimization, 2019
- **[6]** P. Geoffroy Donders. Homogenization method for topology optimization of structures built with lattice materials. Thèse, 2018.