Shape optimization using φ-FEM

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The level-set method for evolving interfaces was first introduced in [7] and has been applied to a variety of shape optimization problems (see e.g. [1]). The core idea of this method is to use a level-set description of the domain to be optimized and modify this level-set e.g. by the derivation of optimality conditions via the shape derivative of the domain.
Like the level-set method, the φ-FEM (first introduced in [4]) takes advantage of a description of the domain via a level-set. The φ-FEM is an immersed boundary finite element method with optimal convergence rate and conditioning which does not require any non-standard quadrature rule on cut cells nor non-standard finite element shape functions [3, 4]. Thus, the φ-FEM is naturally well suited to be used with the level-set method to perform shape optimization as it provides a good description of the evolving boundary without any re-meshing of the domain.
In this contribution, we describe a preliminary study on a shape optimization algorithm based on the level-set method using φ-FEM to solve the state equations. This algorithm is implemented in FEniCSx via the recent FormOpt shape optimization toolbox [5] and the phiFEM python package [2]. We present our method and its application on shape optimization by comparing with a standard non-conforming FEM [6] in different types of shape optimization problems, in two dimensions.

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