Two structure-preserving finite volume schemes for a generalized Poisson–Nernst–Planck system with cross-diffusion

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In this talk, I will present two finite volume approaches [1, 2] for modeling the diffusion of ions in constrained geometries using a degenerate Poisson–Nernst–Planck system with size exclusion, leading to cross-diffusion. Both methods rely on a two-point flux approximation and belong to the class of exponentially fitted schemes. The only difference between them lies in the choice of a Stolarsky mean for the drift term arising from a self-consistent electric potential.

The first version of the scheme, referred to as (SQRA), uses a geometric mean and extends the square-root approximation scheme. The second scheme, (SG), employs an inverse logarithmic mean and yields a generalized version of the Scharfetter–Gummel scheme.

Both schemes are shown to be thermodynamically consistent, in the sense that they ensure the decay of a discrete free energy. Classical numerical analysis results—such as the existence of a discrete solution and the convergence of the scheme as the mesh size and time step tend to zero—are established. The long-time behavior of the schemes is also investigated, both theoretically and numerically. Numerical simulations confirm the theoretical findings, while also indicating a possibly very slow convergence of the system toward equilibrium.

[1] C. Cancès, M. Herda, A. Massimini. *Finite volumes for a generalized poisson-nernst-planck system with cross-diffusion and size exclusion.* In *International Conference on Finite Volumes for Complex Applications*, pp. 57–73. Springer, 2023.

[2] C. Cancès, M. Herda, A. Massimini. *Convergence and long-time behavior of finite volumes for a generalized poisson-nernst-planck system with cross-diffusion and size exclusion.* arXiv preprint arXiv :2411.11583, 2024.

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