Dynamic reordering and quasi-Newton partitioned solvers for strongly coupled thermo-hygro-corrosive PDEs in underground radioactive waste storage

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The French national agency for radioactive waste management (ANDRA) is currently developing CIGÉO, an underground disposal facility based on a steel-lined tunnel network. Over long time scales, corrosion prediction in such structures requires the simulation of strongly coupled thermo-hygro-corrosive partial differential equations, combining heat transfer, water-vapor transport, oxygen transport, and interfacial rust growth. In particular, threshold-type evaporation/condensation phenomena at the air-steel interface make the coupled problem highly nonlinear [2]. In this context, numerical methods must remain accurate enough to preserve strong-coupling effects, while being computationally efficient enough to support large numerical campaigns.

Recent work introduced the *chicken-egg* algorithm, a partitioned quasi-Newton strategy based on dimensionless coupling indicators, which dynamically characterizes cross-physics interaction strengths, reorders partitioned solvers accordingly, and activates synchronization only when required [3]. This strategy was shown to preserve tight-coupling accuracy while significantly reducing redundant synchronization iterations and wall-clock time compared with classical monolithic or diagonalized strategies.

This work presents ongoing developments of this framework in `Cast3M`, a Fortran-based Finite-Element kernel, toward faster quasi-Newton partitioned solvers. The first contribution is a residual-based acceleration inserted inside the thermal module outside synchronization stages, relying on Anderson-type corrections [1] combined with robustness filters. The second contribution is a controlled adaptive reuse of tangent matrices and, in practice, of their direct-solver factorizations, in the spirit of Shamanskii-type modified Newton methods [4], aimed at reducing the dominant cost of repeated linear solves. Current numerical experiments indicate that, in regimes where the residual-based acceleration is effectively triggered, the number of thermal nonlinear iterations and the total wall-clock time are significantly reduced. When combined with adaptive factorization reuse, the gain becomes larger, while monitored temperature norms remain unchanged up to negligible relative differences.

Beyond raw performance, the objective is to design scalable partitioned solvers for industrial multiphysics PDEs, while preparing the next developments of the framework, in particular more generic numerical coupling indicators and future extensions to more complex industrial configurations.

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[3] C. Nahed, J. de Lamare. *Dynamic characterization of cross-physics coupling strengths, a methodology to coupling and reordering partitioned solvers for multiphysics applications.* Computers and Structures, **307**, 107615, 2025. doi : 10.1016/j.compstruc.2024.107615.

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