Structural analysis of high-order centered compact schemes for wave and beam equations

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This work investigates high-order centered compact finite-difference schemes for the spatial approximation of even derivatives. Starting from Taylor expansions, the coefficients of these schemes are characterized through a Vandermonde type linear system. We prove convolution identities and recursive relations that allow higher-order finite-difference formulas to be expressed in terms of lower-order ones.

We apply these relations to the spatial semi-discretization of one-dimensional PDEs involving even-order derivatives, such as the wave equation with homogeneous Dirichlet boundary conditions and the Euler–Bernoulli hinged beam equation. We show that the resulting high-order discretization matrices can be expressed as polynomials of the standard second-order finite-difference Laplacian. Consequently, they share the same eigenvectors as the classical discrete Laplacian, while their eigenvalues can be written explicitly as functions of the underlying discrete spectrum.

The obtained spectral representation allows a precise analysis of the discrete dynamics and of the conserved discrete energies. In particular, it provides a convenient framework for studying spectral gaps and observability properties of semi-discrete systems. These results were obtained in collaboration with N. Cindea and I. Roventa in a work supported by the International Research Center "Innovation Transportation and Production Systems" of the I-SITE CAP 20-25.

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