

Enriching continuous Lagrange finite element approximation spaces using neural networks

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We present a preliminary study [1] combining two approaches in the context of PDE solving : the classical finite element method (FEM) [2] and more recent techniques based on neural networks. Indeed, in recent years, Physics-Informed Neural Networks (PINNs) [3] have become particularly interesting for quickly solving such problems, especially in large dimensions. However, their lack of precision is a major drawback in this context, hence the interest in combining them with FEM, for which error estimators are already known. This combination will make it possible to correct and certify the prediction of neural networks in order to obtain a fast and accurate solution. The complete pipeline proposed here then consists of modifying the classical approximation spaces in FEM by taking the information of a prior, chosen here as the prediction of a PINN. Current results show that pre-processing the problem using neural networks can achieve fixed error targets with coarser meshes than in standard finite element methods, thus saving time in computing the solution. Error estimates have been proven showing that enriched spaces outperform classical ones by a factor that depends only on the quality of the prior.

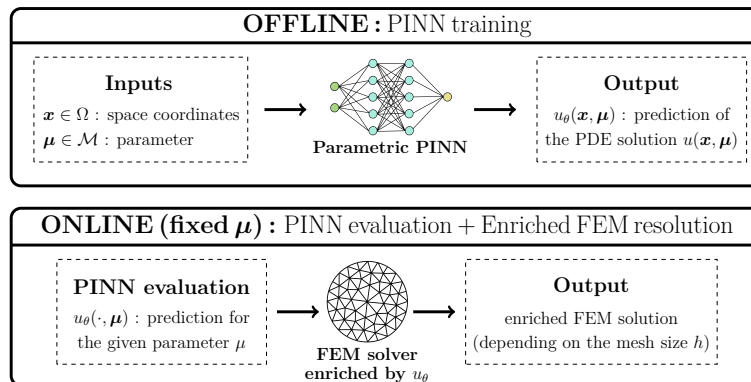


FIGURE 1 – Complete pipeline : offline (top) and online (bottom) processes.

- [1] H. Barucq, M. Duprez, F. Faucher, E. Franck, F. Lecourtier, V. Lleras, V. Michel-Dansac, N. Victorion. *Enriching continuous lagrange finite element approximation spaces using neural networks*, 2026.
- [2] A. Ern, J.-L. Guermond. *Theory and Practice of Finite Elements*. Springer New York, 2004. doi :10.1007/978-1-4757-4355-5.
- [3] M. Raissi, P. Perdikaris, G. E. Karniadakis. *Physics-informed neural networks : A deep learning framework for solving forward and inverse problems involving nonlinear partial differential equations*. J. Comput. Phys., 2019. doi :10.1016/j.jcp.2018.10.045.