Vlasov-Fokker-Planck modeling of relativistic electron beams in storage rings

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Synchrotron radiation facilities produce intense electromagnetic radiation, used as a light source for many scientific experiments and industrial applications. This radiation is emitted by relativistic electron bunches circulating in the storage ring of a particle accelerator. In this talk, I will present joint work with several collaborators on a kinetic model describing the longitudinal dynamics of such a bunch. The model is a Vlasov–Fokker–Planck (VFP) type equation whose self-consistent, nonlocal and non-symmetric interactions account for the wakefield generated by the bunch itself. Above a threshold current, this coupling triggers the micro-bunching instability and the emission of intense coherent THz radiation [3, 2].
The talk will provide an overview of some modeling, analytical and numerical aspects. After describing the derivation of the model, I will present results on the well-posedness of the equation and asymptotic stability of its steady states [4, 1]. I will then discuss ongoing work aimed at designing methods for the direct numerical computation of the periodic orbits that arise in the unstable regime.

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[2] C. Evain, A. A. Diallo, E. Roussel, C. Szwaj, M. Herda, M.-A. Tordeux, F. Ribeiro, M. Labat, N. Hubert, J.-B. Brubach, P. Roy, S. Bielawski. *Synchronization of Synchrotron Radiation Bursts during a spatio-temporal Instability in accelerator-Based source*, 2026.

[3] C. Evain, C. Szwaj, E. Roussel, J. Rodriguez, M. Le Parquier, M.-A. Tordeux, F. Ribeiro, M. Labat, N. Hubert, J.-B. Brubach, et al. *Stable coherent terahertz synchrotron radiation from controlled relativistic electron bunches.* Nature Physics, **15(7)**, 635–639, 2019.

[4] P. Gervais, M. Herda. *Well-posedness and long-time behavior for self-consistent Vlasov-Fokker-Planck equations with general potentials.* Preprint, arXiv :2408.16468 [math.AP] (2024), 2024.

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