Simulation tools for Kerr combs generation in Fabry-Perot resonators

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Kerr frequency combs refer to a laser source whose spectrum consists of a series of discrete, equally spaced frequency lines, which are generated in an optical resonator from a continuous wave pump laser by Kerr nonlinearity. Our study concerns Fabry-Perot (FP) resonators formed by an optical fiber (centimeters long) bounded at each end by multi-layer dielectric mirrors.

A model to simulate light-wave propagation in a FP resonator in the context of Kerr comb generation relies on the Lugiato–Lefever equation (LLE) with an unknown ψ related to the slowly varying electric field envelope. This nonlinear Schrödinger equation with damping, detuning, and driving force reads

$$\frac{\partial \psi}{\partial t}(z,t) = -\mathrm{i}\frac{\beta}{2}\,\frac{\partial^2 \psi}{\partial z^2}(z,t) - (1+\mathrm{i}\alpha)\,\psi(z,t) + \mathrm{i}\,\psi(z,t)\left(|\psi(z,t)|^2 + \frac{1}{L}\int_{-L}^{L} |\psi(\zeta,t)|^2\,\mathrm{d}\zeta\right) + F \qquad (1)$$

where $z \in]-L, L[$ refers to the space variable, t is a local time and the constant unit-less parameters are $F > 0$ the amplitude of the laser pump, β the cavity group velocity dispersion (GVD) parameter and α the cavity phase detuning of the laser pump. The LLE (1) is to be considered together with periodic boundary and initial conditions.

We have developed several numerical tools to simulate Kerr comb generation in FP resonator by solving LLE (1) using different numerical approaches. A Split-Step method has been implemented to solve the LLE (1) whose efficiency relies on the exact solving of both the linear and the non-linear problems involved in the splitting [2]. In practice, LLE solutions are strongly sensitive to the initial condition and only solutions that become stationary give rise to frequency combs. Thus, we have also implemented a collocation method to solve the stationary LLE [3]. Finally, to mimic the experimental way frequency combs are caught by tuning some of the parameters, we have used a pseudo-arc-length numerical continuation method from the onsets of bifurcations branches on the trivial branch of solutions, with either F or α as bifurcation parameter. Details on these simulation tools and the underlying numerical methods can be found in [1].

A point of practical interest for physicists is the effects of small fluctuations of the amplitude of the laser pump, and more generally the effects of environmental noise, on the intrinsic characteristics of the frequency combs. Such a study which requires a large amount of simulation data for a statistical analysis has led us to resort to parallel computing. Namely, using Fortran for the core numerical implementation together with MPI to distribute the computational workload across multiple processors, significantly accelerates the numerical simulations and enables efficient large-scale statistical analyses. The simulations were executed in either sequential or parallel mode on the GRID5000 computing infrastructure (`www.grid5000.fr`).

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[1] M. Al Sayed Ali, *et al. Numerical simulation of Lugiato-Lefever equation for Kerr combs generation in Fabry-Perot resonators*, 2025, hal.science/hal-04880490.

[2] S. Balac. *S3F4LLE : a Matlab solver for the Lugiato-Lefever equation in the dynamic regime*, 2022, hal.science/hal-04662404.

[3] S. Balac. *COLLE : a Matlab toolbox for solving the Lugiato-Lefever equation in the stationary regime*, 2023, hal.science/hal-04318809.

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