Strong stability of characteristic discretizations of IBVP: the case of lattice Boltzmann schemes

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We study the stability of one-dimensional linear lattice Boltzmann schemes for scalar hyperbolic equations with respect to boundary data [1]. Our approach is based on the original raw algorithm that, although addressing a scalar PDE, features several unknowns. We thus avoid the need for a transformation into an equivalent multi-step scalar formulation. We introduce two distinct notions of strong stability, one of which accounts for the potential absence of a continuous extension of the stable vector bundle associated with the bulk scheme on the unit circle for certain components of the solution vector. Rather than developing a general theory, complicated by the fact that discrete boundaries in lattice Boltzmann schemes are inherently characteristic, we focus on strong stability–instability for methods whose characteristic equations feature stencils of breadth one to the left. In this context, we study representative schemes endowed with various boundary conditions from the literature, and our theoretical results are supported by numerical simulations.

[1] T. Bellotti. *Stability of lattice Boltzmann schemes for initial boundary value problems in raw formulation.* ESAIM : Mathematical Modelling and Numerical Analysis, **60(1)**, 143–195, 2026.

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