Generating functions for variational integrators

Oscar COSSERAT, Instituto de Ciencias Matemáticas - Madrid

In classical mechanics, the dynamics is provided by a Lagrangian whose extremal paths verify the Euler-Lagrange equation. This talk will explain how Lagrangian submanifolds are a fruitful tool to compute dynamics coming from a variational principle.
At first, I will explain the finite dimensional case, where the geometrical setting is clear. There, I will use some toy examples to illustrate what kind of Lagrangian submanifolds appear and how do they relate to the invariant theory of symplectic numerical methods.
In a second part of the talk, I will sketch some approaches to the infinite-dimensional case. Indeed, partial differential equations deriving from a Lagrangian are at the same time much more intricate and contain many interesting and challenging dynamics. A long term project is to develop such methods to mechanical contexts such as elasticity, where differential geometry may lead to the development of accurate numerical methods delivering, in turn, new mechanical insights.
See, e.g., [1] for a book on generating functions, [2] for some invariant theory of symplectic numerical methods and [3] for some variational integrators in field theory. This talk is based on a work in progress with D. M. de Diego, S. Leyendecker, R. Sato and T. Wang.

[1] S. Benenti. *Hamiltonian Structures and Generating Families.* Universitext. Springer New York, 2011.

[2] O. Cosserat. *Some stability properties of Hamiltonian Poisson integrators*, 2025.

[3] J. Vankerschaver, C. Liao, M. Leok. *Generating functionals and lagrangian partial differential equations.* Journal of Mathematical Physics, **54(8)**, 082901, 2013. doi :10.1063/1.4817391.

Contact : `oscar.cosserat@icmat.es`