Modelling and contact control of slender locomotors - Applications to bio inspired robotics

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Slender locomotors have been the subject of increasing attention in recent years due to their remarkable energetic efficiency and adaptability to complex environments. These systems achieve locomotion through complex gaits enabled by their large number of degrees of freedom, which raises important challenges both in robotics and mathematics. In this poster, we present a mathematical model of such locomotors and derive conditions on the number and the type of controls required to control the system. We are primarily interested in Small-Time Local Controllability (STLC), that is whether the system can be steered locally in any direction in arbitrary small time.
Although slender locomotors have been widely studied, a mathematical framework capable of capturing and explaining the complex gaits observed in nature has yet to be fully developed, particularly regarding contact modeling. Indeed, existing models in literature are often developed from a numerical perspective, tailored to a specific robot or using shape-based controls [1]. Despite its recognized importance in related problems such as micro-swimmer dynamics [2], elasticity is frequently neglected, and positive STLC results are still lacking, even in discrete models [3].
Slender locomotors are typically modeled as high-dimensional nonlinear control systems, with finite-dimensional models taking the form of an affine control system with drift :

$$\dot{q} = f_0(q) + \sum_{i=1}^{m} u_i(t) f_i(q), \quad u(t) = (u_i(t))_{1\leq i\leq m} \in \Omega \tag{Aff.}$$

where we denote $q \in \mathbb{R}^d$ the state of the control system, $f_0 \in \mathcal{C}^\infty(\mathbb{R}^d, \mathbb{R}^d)$ the drift vector field, $(f_i)_{1\leq i\leq m} \in \mathcal{C}^\infty(\mathbb{R}^d, \mathbb{R}^d)^m$ the control vector fields, $\Omega \subset \mathbb{R}^m$ the set of control constraints and $u \in L^\infty([0,T], \Omega)$ the control. The main tools characterizing the STLC property of (Aff.) are Lie brackets, which capture the non-commutativity between the vector fields $(f_i)_{0\leq i\leq m}$. While there exist some sufficient conditions for establishing STLC of (Aff.) (such as the celebrated Sussmann condition), sharp necessary conditions are known only for $m = 1, 2$.
Considering a locomotor moving in a plane, modeled as a discrete system of n elastic beams, we use Lagrangian mechanics to derive a control system of the form (Aff.) governing the locomotor's motion. This model notably incorporates internal control in torques and forces, contact control and stiffness (both angular and linear). We present a detailed study of the controllability of the system in the inertia-free case with $n = 2, 3$. The system being not STLC using only contact control, we review what types of internal controls allow us to recover STLC. We also highlight the importance of anisotropic friction in the case where where only internal controls are available. We conclude by outlining several perspectives we intend to address in our future work.

[1] F. Boyer, S. Ali, M. Porez. *Macrocontinuous dynamics for hyperredundant robots : application to kinematic locomotion bioinspired by elongated body animals.* IEEE Transactions on Robotics, **28(2)**, 303–317, 2011.

[2] K. Ishimoto, J. Herault, C. Moreau. *Bending–compression coupling in extensible slender microswimmers.* Journal of Fluid Mechanics, **1020**, A1, 2025.

[3] P. Liljeback, K. Y. Pettersen, Ø. Stavdahl, J. T. Gravdahl. *Controllability and stability analysis of planar snake robot locomotion.* IEEE Transactions on Automatic Control, **56(6)**, 1365–1380, 2010.

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