# An efficient second-order positivity-preserving IMEX finite volume scheme for compressible flows on staggered meshes.

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In this work, we present a positivity-preserving quasi-second order in space and time finite volumes scheme on staggered meshes for Navier-Stokes equations with simplified viscosity terms. This is an on-going work that will be used by the LIE team at the Autorité de Sûreté Nucléaire et de Radioprotection (ASNR) in a LES solver for turbulence simulations.
The space discretization is a standard Finite Volume Method (FVM) on staggered meshes, and the time-stepping strategy is a 4-stages IMplicit-EXplicit (IMEX) Runge-Kutta method.
The inviscid part (the Euler equations) are discretized explicitely in time *via* a 4-stages Segregated Runge-Kutta (SRK) scheme. It is positivity preserving for the density and internal energy thanks to a flux limitation in-between intermediate Runge-Kutta steps derived from [1]. The limiter used is the Algebraic-MUSCL (A-MUSCL) algorithm [2]. It computes quasi-second order positivity preserving inter-cell values by projecting high-order (not positivity preserving) inter-cell approximations on a stability interval given by the values of adjacent cells. It is an easy-to-implement limiter that works well regardless of the space dimension.
The viscous terms are discretized implicitely in time *via* a 2-steps non-standard Diagonally Implicit Runge-Kutta method with first Explicit step (EDIRK), where even the last intermediate step has an implicit part. Numerical experiments show this helps reduce (even nullify) the stringent parabolic CFL associated to the explicit part of the EDIRK method. The viscous and diffusion terms are taken into account in only half the intermediate stages of the SRK method, reducing the computing costs associated to solving large systems.
This convoluted discretization yields the following benefits :

- positivity-preserving scheme,
- second-order in space and time,
- reduced computational cost compared to standard IMEX schemes.

One current issue of the scheme is a lack of stability when diffusion and visosity are very small.

[1] A. Ern, J.-L. Guermond. *Invariant-domain-preserving high-order time stepping : I. imex schemes.* SIAM Journal on Scientific Computing.

[2] L. Piar, F. Babik, R. Herbin, J.-C. Latché. *A formally second order cell centered scheme for convection-diffusion equations on unstructured non-conforming grids.* International Journal for Numerical Methods in Fluids.

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