# On aromatic trees and related algebraic structures for the study of volume-preserving numerical integrators

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Aromatic Butcher series are the natural extension of the standard B-series that allows to compute the divergence of a Taylor expansion over the jet space of a given ODE. First used as a tool for the study of volume-preserving numerical integrators, they were soon studied for their numerous algebraic properties. In this talk, we will present an overview of recent results, both numerical and algebraic, concerning aromatic trees. After an intuitive definition of aromas with an invariant tensor theorem, we introduce the aromatic bicomplex [5], in the spirit of the variational bicomplex in variational calculus. This new object proves crucial for characterising Ker(Div) on aromatic trees, derive negative results on volume-preserving methods, and formulate the notion of symmetries and Noether theorems [4] for tree-like structures. If time allows, we will give an overview of related algebraic structures, that are aromatic multi-indices [6], Hopf algebroids [1, 2], and planar aromatic trees [3], and discuss how they will be used to tackle the open problems of geometric integration concerning volume-preservation.

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[3] A. Busnot Laurent, H. Munthe-Kaas, G. S. Venkatesh. *The free tracial post-Lie-Rinehart algebra of planar aromatic trees for the design of divergence-free Lie-group methods.* Submitted, 2026.

[4] A. Laurent. *The Lie derivative and Noether's theorem on the aromatic bicomplex for the study of volume-preserving numerical integrators.* J. Comput. Dyn., **11(1)**, 10–22, 2024. doi : 10.3934/jcd.2023011.

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