Periodic solution computation for spatio-temporal conservative systems: Application to the 1D Mckean-Vlasov model

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In this work, we investigate the application of classical Newton and Newton–Picard[3] methods to the computation of periodic solutions of temporal or spatio-temporal systems. For conservative systems—those possessing one or more first integrals, the standard formulations of these methods do not inherently preserve the relevant invariants, and periodic orbits typically form families parametrized by conserved quantities. Building on the continuation framework for periodic orbits in conservative dynamical systems developed in [4], we apply these ideas to incorporate conserved quantities directly into Newton- and Newton–Picard-type iterations, treating linear invariants (e.g., mass) explicitly.
The resulting mass-preserving algorithms are used to compute a family of periodic solutions and a bifurcation diagram of the McKean–Vlasov model, which, as the overdamped limit of the Vlasov–Fokker–Planck equation [1], provides a fundamental mean-field setting for developing invariant-preserving continuation methods relevant to electron bunch dynamics in particle accelerators[2].

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[3] K. Lust, D. Roose, A. Spence, A. R. Champneys. *An Adaptive Newton-Picard Algorithm with Subspace Iteration for Computing Periodic Solutions.* SIAM Journal on Scientific Computing, **19(4)**, 1188, 1998.

[4] F. J. Muñoz-Almaraz, E. Freire, J. Galán, E. Doedel, A. Vanderbauwhede. *Continuation of periodic orbits in conservative and Hamiltonian systems.* Physica D : Nonlinear Phenomena, 2003.

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