Green Waves and Homogenization for a Hamilton-Jacobi Traffic Flow Model

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Traffic flow is classically described by the LWR model, a scalar conservation law that can be written in an equivalent Hamilton-Jacobi form. In this presentation, we consider a traffic flow model posed on a single infinite road with a large number of traffic lights. In this setting, a green wave may occur when the traffic lights are synchronized so that cars can pass through a sequence of green lights without stopping. Motivated by this mesoscopic problem, we study a Hamilton-Jacobi equation posed on the infinite real line, with traffic lights equally spaced along the road. The traffic lights follow the same periodic signal, with a fixed phase shift in time between consecutive lights.
At the mesoscopic level, we model the effect of each traffic light individually, and we describe the traffic between consecutive lights in terms of the car density. The goal is to look at this model from far away in order to obtain a macroscopic description. In this description, the effect of the individual lights is replaced by an averaged effect on the traffic density. The mathematical theory used to pass from the first description to the second is homogenization. This leads to an effective Hamilton-Jacobi equation, whose effective Hamiltonian (or effective flux) describes the large-scale behavior of traffic and keeps the memory of the mesoscopic effect of the traffic lights.
Unlike many homogenization results, a key feature of our work is that the effective flux can be computed explicitly. Using an optimal control approach, we indeed derive an explicit formula. This makes it possible to see directly how the phase shift between consecutive traffic lights affects the effective flux and, in turn, the overall traffic flow. When the original Hamiltonian corresponds to a triangular fundamental diagram, we can even determine the phase shift that gives rise to a green wave. Our results also reveal that, to obtain a green wave, the phase shift should depend on the traffic regime : we exhibit one phase shift for free-flow traffic and another one for congested traffic.
Finally, we present numerical experiments illustrating the behavior of the effective flux and its dependence on the phase shift.

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