

Lyapunov functions and stability of linear two-dimensional hyperbolic systems

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We study the boundary stability and stabilization of two-dimensional linear hyperbolic systems. In contrast with the 1-D case, intensively studied in the last forty years, in multi-dimensional hyperbolic systems, a basic quadratic Lyapunov function for the L^2 norm is usually a weighted L^2 norm, with weights that must be uniform across all components.

We show that, contrarily to the 1-D case, these Lyapunov functions, can only prove stability of multi-dimensional systems that already possess a specific internal structure. This drastically limits the applicability of this impressive tool, introduced by Bastin and Coron, in the multi-dimensional case. We demonstrate that this limitation can be overcome by considering a different sort of quadratic Lyapunov functions for higher norm, such as H^1 norm, thereby opening up the possibility of finding quadratic Lyapunov functions for systems that do not satisfy a dissipative structure. We successfully construct a quadratic Lyapunov function for the H^1 norm for a concrete example, confirming the feasibility of our approach. Furthermore, we show that these quadratic Lyapunov function for the H^1 norm can be used to construct a Lyapunov function for the L^2 norm, thereby establishing L^2 stability for such systems.