Residual based a posteriori error analysis for φ-FEM

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The φ-FEM [3] is an immersed boundary finite element method taking advantage of the description of the exact geometry via a level-set. Unlike the popular methods XFEM [4] and CutFEM [2], φ-FEM needs no unusual shape functions or quadrature rules near the boundary of the domain. Leveraging the level-set description of the boundary makes φ-FEM easy to implement while preserving an optimal convergence rate and conditioning.
In this talk, we introduce a novel a posteriori error estimator for φ-FEM. Our estimator is composed of a standard residual part [5] and is augmented by boundary related terms used to estimate the errors coming from the discretization of the boundary condition and the geometry. We show that this estimator is both reliable and efficient with respect to a measure of the discretization error including the boundary approximation error and give the key ideas of the proofs.
This estimator and an adaptive mesh refinement strategy is implemented in FEniCSx [1]. We investigate their performance on several numerical test cases with various geometries. In particular, we highlight an improvement of the boundary approximation when the boundary related terms of the estimator are included.

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