Neural Network approximation of the Mortensen observer in high-dimension

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In this preliminary work, we address the numerical resolution of nonlinear filtering problems by proposing a scalable neural network algorithm for the approximation of a nonlinear observer.

Estimating the state of an finite-dimensional system from partial noisy observations is a classical problem known as the filtering problem. In the linear dynamics and linear observations framework, this problem was solved by Kalman and Bucy in their classical work [1].

For nonlinear dynamics, the analog of the Kalman filter is called the Mortensen observer [3].

The Mortensen observer is defined as the minimum least-squares estimator of the state at a given time. An equivalent approach is to see it as the instantaneous minimizer of the *value function* associated with the optimal control problem one obtains by interpreting the noise of the filtering problem as a *control* input acting on the dynamics.

The value function is known to satisfy a nonlinear Hamilton-Jacobi-Bellman (HJB) equation, so that discretizing the latter allows for an approximation of the Mortensen observer.
Adhering to the discretize-then-optimize philosophy, we consider a discrete analog of the dynamics and tackle the filtering problem by *controlling the discrete system.* When the discrete dynamic system is consistent with the time-continuous one, a time-splitting scheme for the value function has been proposed in [2], that is proved to be convergent in the vanishing time-step limit.

Based on this semi-discretization, we propose to build a numerical approximation by supplementing the time-splitting scheme with an Extended Kalman Filter (EKF)-based algorithm using Input Convex Neural Networks. This algorithm is less sensitive to the curse of dimensionality than traditional grid-based HJB techniques, as will be illustrated numerically in dimension $d \gg 1$, where grid-based methods become intractable.

[1] R. E. Kalman, R. S. Bucy. *New results in linear filtering and prediction theory.* Trans. ASME Ser. D. J. Basic Engrg., **83**, 95–108, 1961.

[2] P. Moireau. *A discrete-time optimal filtering approach for non-linear systems as a stable discretization of the Mortensen observer.* ESAIM Control Optim. Calc. Var., **24(4)**, 1815–1847, 2018. doi:10.1051/cocv/2017077.

[3] R. E. Mortensen. *Maximum-likelihood recursive nonlinear filtering.* J. Optim. Theory Appl., **2(6)**, 386–394, 1968. doi:10.1007/BF00925744.

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