Learning the regularisation of a singular closure for the five-moment system

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The kinetic modelling of rarefied gases via the Boltzmann or BGK equations gives rise to an infinite hierarchy of velocity moments, whose truncation requires an algebraic closure for the highest-order moment. For the five-moment system in one space and one velocity dimension, this closure reduces, after Galilean reduction and non-dimensionalisation, to a function of two normalised variables on an explicit realizability domain.

All classical closures suffer from structural defects : Grad's closure loses hyperbolicity as soon as one departs from equilibrium, while the maximum-entropy closure is implicit and singular on a half-line including states arbitrarily close to equilibrium. The explicit family introduced by [1], parametrised by a regularisation scalar $\beta > 0$, interpolates between these two limits while preserving realizability, Galilean invariance and hyperbolicity ; no analytical criterion, however, selects an optimal value of β for a given flow regime.

We propose to learn the regularisation as a state-dependent function $\beta_\theta(\hat{Q}, \hat{R})$ of the local moment state, parametrised by a small neural network and trained on kinetic reference data. The resulting closure inherits, by construction, all the structural properties of the Schaerer–Torrilhon family, and a spectral penalty on the imaginary parts of the flux-Jacobian eigenvalues enforces hyperbolicity during training. Preliminary numerical experiments on a two-stream Riemann problem suggest that the network spontaneously identifies the regions of phase space where the regularisation must be strengthened, and yields a closure that uniformly improves over any fixed-β choice on the realizability domain.

[1] R. P. Schaerer, M. Torrilhon. *On singular closures for the 5-moment system in kinetic gas theory.* Communications in Computational Physics, **17(2)**, 371–400, 2015.

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