Nonlinear preconditioning techniques for unbalanced nonlinear systems

Quang Huy TRAN, IFPEN - Paris
Alessandra MARELLI, IFPEN, Université Côte d'Azur, INRIA Sophia-Antipolis - Nice
Konstantin BRENNER, Université Côte d'Azur - Nice

The numerical discretization of nonlinear partial differential equations, such as the Richards equation, results in large algebraic nonlinear systems of equations. These systems are typically solved using Newton's method, which exhibits quadratic convergence under certain assumptions on the initial point and the residual function. Additionally, under the further conditions of concavity (or convexity) of the residual function and nonnegative inverse of its Jacobian, Newton's method ensures global monotone convergence. For nonsmooth problems, the semismooth Newton method extends these results under analogous conditions. Despite these favorable properties, Newton's method can encounter significant computational challenges, leading to a deterioration in performance. In this context, nonlinear preconditioning techniques play a crucial role in enhancing its effectiveness. Here, we focus on two approaches based on domain decomposition : the iterative NlEm (Nonlinear Elimination method), a multiplicative strategy that targets a subset of degrees of freedom associated with strong nonlinearities through local elimination, and a block Jacobi/Newton method, an additive technique analogous to the known NKS-RAS method. In this work, we investigate how global monotone convergence properties are affected by nonlinear preconditioning techniques. For both methods, we prove that global monotone convergence is preserved under the additional assumption that the (generalized) Jacobian satisfies an M-matrix property. The proposed framework is applied to three representative problems : the Richards equation, the porous medium equation, and the obstacle problem. Numerical experiments demonstrate that these nonlinear preconditioning strategies significantly improve the robustness and efficiency of Newton-type methods, particularly for problems with unbalanced or stiff nonlinearities, while maintaining stability with respect to physical parameters and mesh refinement.

Contact : alessandra.marelli@univ-cotedazur.fr