Event-triggered control and observer design for infinite-dimensional systems

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Outline

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Motivation

- A control system is a dynamical system on which we can **act by using suitable controls** (closed-loop) which (asymptotically) **stabilises state** we want to reach

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  $\Rightarrow$ *real-time simulation* of system driven by same input
  $\Rightarrow$ correction derived from difference between actual output and predicted output
- Our goal is to **investigate stabilisation** of abstract infinite-dimensional (PDE) control system **using Luenberger observer and event-triggered updates of control**

In a nutshell

- Linear systems governed by ODE, matrices A, B, state $x(\cdot)$, control input $u(\cdot)$

$$\dot{x}(t) = Ax(t) + Bu(t), \quad t \geq 0, \tag{1}$$

[1] Tabuada 2007; Heemels, Johansson, and Tabuada 2012; Donkers and Heemels 2012.

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- In ETC[1], control input is updated *only* at discrete instants $(t_k)_{k\in\mathbb{N}}$

$$\begin{aligned} &\dot{x}(t) = Ax(t) + Bu(t_k), \quad t \in [t_k, t_{k+1}), \quad k \in \mathbb{N}, \\ &\|x(t_k) - x(t)\|^2 < \sigma\|x(t)\|^2, \quad t \in [t_k, t_{k+1}), \quad \sigma \in (0,1), \end{aligned} \tag{2}$$

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- Transition to PDE is far from trivial[3] due to unbounded operators, semigroup dynamics, coexistence of stable and unstable spectral modes, etc

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- Well-known[4] that if ($\mathcal{H}$ and $\mathcal{U}$ Hilbert spaces)
  1. $A : \mathcal{D}(A) \subset \mathcal{H} \to \mathcal{H}$ is infinitesimal generator of a C_0-semigroup $\exp(tA)$
  2. $B \in \mathcal{L}(\mathcal{U}, \mathcal{H})$ and $K \in \mathcal{L}(\mathcal{H}, \mathcal{U})$ are bounded linear operators
  3. $A + BK$ is infinitesimal generator of an exponentially stable C_0-semigroup on $\mathcal{H}$
  4. A is skew-adjoint (i.e. $A^* = -A$)

then for $z_0 \in \mathcal{D}(A)$ closed-loop system (3) is **exponentially stable**, i.e.

$$\exists C_0 > 0, \quad \exists \lambda > 0, \quad \|z(t)\|_{\mathcal{H}}^2 \leq C_0 e^{-\lambda t}, \quad \forall t \geq 0, \tag{4}$$

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Continuous closed loop system (observation-based control)

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- Assuming that ($\mathcal{H}$, $\mathcal{U}$ and $\mathcal{Y}$ Hilbert spaces)
  1. $A : \mathcal{D}(A) \subset \mathcal{H} \to \mathcal{H}$ is infinitesimal generator of a C_0-semigroup $\exp(tA)$
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  3. $C \in \mathcal{L}(\mathcal{H}, \mathcal{Y})$ is a bounded linear operator
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  5. A is skew-adjoint

then for $z_0 \in \mathcal{D}(A)$ closed-loop system (5) is **exponentially stable**, i.e.

$$\exists C_0 > 0, \quad \exists \lambda > 0, \quad \|z(t)\|^2_{\mathcal{H}} \leq C_0 e^{-\lambda t}, \quad \forall t \geq 0, \tag{6}$$

ETC design

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- Control strategy where updates happen only when a certain event occurs
- Event is usually defined by a threshold, on system state or error
- **Reduces unnecessary communication** and computation compared with periodic control (periodic control acts on a schedule while ETC acts only when needed)

- LB, SE[5] established exponential stability of control system (3) when control is based on an observation operator and subjected to aperiodic sample-and-hold mechanism

$$\begin{cases} \dot{z}(t) = Az(t) + Bu(t_k) & t \in [t_k, t_{k+1}), \quad k \in \mathbb{N}, \\ u(t) = Ky(t), & t \geq 0, \\ y(t) = Cz(t), & t \geq 0, \\ z(0) = z_0, \end{cases} \tag{7}$$

where $z_0 \in \mathcal{H}$, subjected to triggering mechanism

$$t_{k+1} := \sup \left\{ t > t_k, \quad \|Cz(t) - Cz(t_k)\|_{\mathcal{H}} \leq \gamma \|z(t)\|_{\mathcal{H}} \right\}, \quad \gamma > 0, \tag{8}$$

[5] Baudouin and Ervedoza 2025.

ETC system

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  then for $z_0 \in \mathcal{D}(A)$ ETC system (7)-(8) is **exponentially stable** i.e.

$$\exists C_0 > 0, \quad \exists \lambda > 0, \quad \|z(t)\|^2_{\mathcal{H}} \leq C_0 e^{-\lambda t}, \quad \forall t \geq 0, \tag{9}$$

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- State-feedback control requires value (at each time) that **approximates**[6] system state

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Observer design

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- **Hardware sensors** are physical instruments that directly measure state variables
  ⇒ sometimes expensive, difficult to implement, or unable to measure all state

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Observer design

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- **Hardware sensors** are physical instruments that directly measure state variables
  $\Rightarrow$ sometimes expensive, difficult to implement, or unable to measure all state
- **Software sensors** algorithms *based on a model and available physical measurements*
  $\Rightarrow$ provide online estimates of unmeasured state and implementable in practice

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- Observer design control system

$$
\begin{cases}
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then for $(z_0, \hat{z}_0) \in \mathcal{D}(A) \times \mathcal{D}(A)$ control system (10) is **exponentially stable**, i.e.

$$\exists C_0 > 0, \quad \exists \lambda > 0, \quad \|z(t)\|^2_{\mathcal{H}} + \|\hat{z}(t)\|^2_{\mathcal{H}} \leq C_0 e^{-\lambda t}, \quad \forall t \geq 0, \tag{11}$$

- ETC observer design control system

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then for $(z_0, \hat{z}_0) \in \mathcal{D}(A) \times \mathcal{D}(A)$ **IS** control system (12) **exponentially stable ?**, i.e.

$$\textbf{?}\, (\exists C_0 > 0, \quad \exists \lambda > 0), \quad \|z(t)\|^2_{\mathcal{H}} + \|\hat{z}(t)\|^2_{\mathcal{H}} \leq C_0 e^{-\lambda t}, \quad \forall t \geq 0, \tag{14}$$

Dynamic triggering rule

- To design triggering mechanism that determines $(t_k)_{k\in\mathbb{N}}$, can not consider

$$t_{k+1} := \sup\left\{t > t_k, \quad \|\hat{z}(\tau) - \hat{z}(t_k)\|_{\mathcal{H}} \leq \gamma\|\mathbf{z}(\tau)\|_{\mathcal{H}} + \gamma\|\hat{z}(\tau)\|_{\mathcal{H}}, \quad \forall\tau \in [t_k, t)\right\}, \tag{15}$$

since it requires knowledge of **true state** $z(\cdot)$ **assumed unmeasured**

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- *To increase flexibility of triggering mechanism* and to **achieve better control of inter-event times**, we equip system (12) with internal scalar dynamic[7] variable $m(\cdot)$

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- Introduce maximal time T^* under which system (12)-(17) has a solution

$$T^* := \begin{cases} \infty, & \text{if card}\left((t_k)_{k\in\mathbb{N}}\right) < \infty, \\ \limsup_k t_k, & \text{otherwise} \end{cases} \tag{18}$$

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Key assumptions

$\mathcal{H}$, $\mathcal{U}$ and $\mathcal{Y}$ are Hilbert spaces

Assumptions (G)

- $A : \mathcal{D}(A) \subset \mathcal{H} \to \mathcal{H}$ *is infinitesimal generator of a* C_0*-semigroup* $\exp(tA)$
- $B \in \mathcal{L}(\mathcal{U}, \mathcal{H})$ *and* $K \in \mathcal{L}(\mathcal{H}, \mathcal{U})$ *are bounded linear operators*
- $A + BK$ *is infinitesimal generator of an exponentially stable* C_0*-semigroup on* $\mathcal{H}$
- $C \in \mathcal{L}(\mathcal{H}, \mathcal{Y})$ *and* $L \in \mathcal{L}(\mathcal{Y}, \mathcal{H})$ *are bounded linear operators*
- $A + LC$ *is infinitesimal generator of an exponentially stable* C_0*-semigroup on* $\mathcal{H}$
- *Generator* A *satisfies quasi-dissipativity condition*

$$\exists c_A \geq 0, \quad \forall z_0 \in \mathcal{D}(A), \quad \mathrm{Re} \langle Az_0, z_0 \rangle_{\mathcal{H}} \leq c_A \|z_0\|^2_{\mathcal{H}} . \tag{19}$$

Assumption (M)

- $m \in C^0(\mathbb{R}_+; \mathbb{R}_+)$,
- $\forall T > 0, \quad \exists \delta_T > 0, \quad \inf_{t \in [0,T]} m(t) \geq \delta_T,$
- $\exists C_m > 0, \quad \exists \mu > 0, \quad m(t) \leq C_m e^{-\mu t}, \quad \forall t \geq 0,$

Augmented state $Z = (\hat{z}, e)^\top \in \mathcal{H} \times \mathcal{H}$ satisfies

$$\begin{cases} \dot{Z}(t) = \mathcal{A}_{\mathrm{aug}} Z(t) + \hat{d}_k(t), & t \in [t_k, t_{k+1}), \quad k \in \mathbb{N}, \\ Z(0) = Z_0, \end{cases} \tag{20}$$

$$\mathcal{A}_{\mathrm{aug}} := \begin{pmatrix} A + BK & LC \\ 0 & A + LC \end{pmatrix}, \quad \mathcal{D}(\mathcal{A}_{\mathrm{aug}}) = \mathcal{D}(A) \times \mathcal{D}(A), \tag{21}$$

$$\hat{d}_k(t) := \begin{pmatrix} BK(\hat{z}(t_k) - \hat{z}(t)) \\ 0 \end{pmatrix}, \quad t \in [t_k, t_{k+1}), \quad k \in \mathbb{N}, \tag{22}$$

Lemma (Augmented closed-loop generator)

Under Assumptions (G), $\mathcal{A}_{\mathrm{aug}}$ generates a C_0-semigroup on $\mathcal{H} \times \mathcal{H}$.

Well-posedness

Theorem (Well-posedness of a maximal solution)

Under Assumptions (G) and Assumption (M), there exist a strictly increasing sequence $(t_k)_{k\in\mathbb{N}} \subset \mathbb{R}_+$ with $t_0 = 0$ and a **unique mild solution**

$$Z \in C([0, T^*); \mathcal{H} \times \mathcal{H}), \tag{23}$$

where $T^* = \sup_k t_k \in (0, \infty]$, such that

$$Z(t) = \exp((t - t_k)\mathcal{A}_{\text{aug}})Z(t_k) + \int_{t_k}^{t} \exp((t - s)\mathcal{A}_{\text{aug}}) \begin{pmatrix} BK\hat{z}(t_k) \\ 0 \end{pmatrix} ds, \quad \forall t \in [t_k, t_{k+1}), \tag{24}$$

and triggering rule (17) holds on each interval $[t_k, t_{k+1})$ for $k \in \mathbb{N}$,

Theorem (Piecewise classical regularity)

Under Assumptions (G) *and Assumption* (M), *and if in addition* $Z_0 \in \mathcal{D}(A) \times \mathcal{D}(A)$ *then* $Z(\cdot)$ *is* **weakly continuous** *in* $[0, T^*)$ *with values in* $\mathcal{D}(A) \times \mathcal{D}(A)$ *and* **piecewise classical** *in* $[0, T^*)$ *with values in* $\mathcal{H} \times \mathcal{H}$, *i.e.*

$$Z \in C_w((t_k, t_{k+1}); \mathcal{D}(A) \times \mathcal{D}(A)) \cap C^1((t_k, t_{k+1}); \mathcal{H} \times \mathcal{H}), \quad \forall k \in \mathbb{N}. \tag{25}$$

Moreover, it holds that for all $t \in [0, T^*)$,

$$\left\|\dot{\hat{z}}\right\|_{L^\infty(0,t;\mathcal{H})} \leq e^{(\|BK\|+c_A)t} \left\|(A + BK)\hat{z}_0 + LCe_0\right\|_{\mathcal{H}} + te^{(\|BK\|+c_A)t} \left\|LC\right\| M_0 \left\|(A + LC)e_0\right\|_{\mathcal{H}}, \tag{26}$$

Theorem (No zeno)

*Under Assumptions (G) and Assumption (M), solution to system (**??**) subjected to triggering rule (17) satisfies*

$$T^* = \infty, \tag{27}$$

and thus does **not experience any Zeno phenomenon**

Theorem (Event-triggered stability)

Choosing small enough γ, under Assumptions (G)-(M), then event-triggered system (12)-(17) is **exponentially stable** *in $\mathcal{H} \times \mathcal{H}$ for an initial datum in $\mathcal{D}(A) \times \mathcal{D}(A)$, i.e.*

$$\exists \hat{C} > 0, \quad \exists \hat{\lambda} > 0, \quad \|\hat{z}(t)\|_{\mathcal{H}}^2 + \|e(t)\|_{\mathcal{H}}^2 \leq \hat{C} \left(\|\hat{z}_0\|_{\mathcal{H}} + \|e_0\|_{\mathcal{H}} + m_0 \right) e^{-\hat{\lambda} t}, \quad \forall t \geq 0, \tag{28}$$

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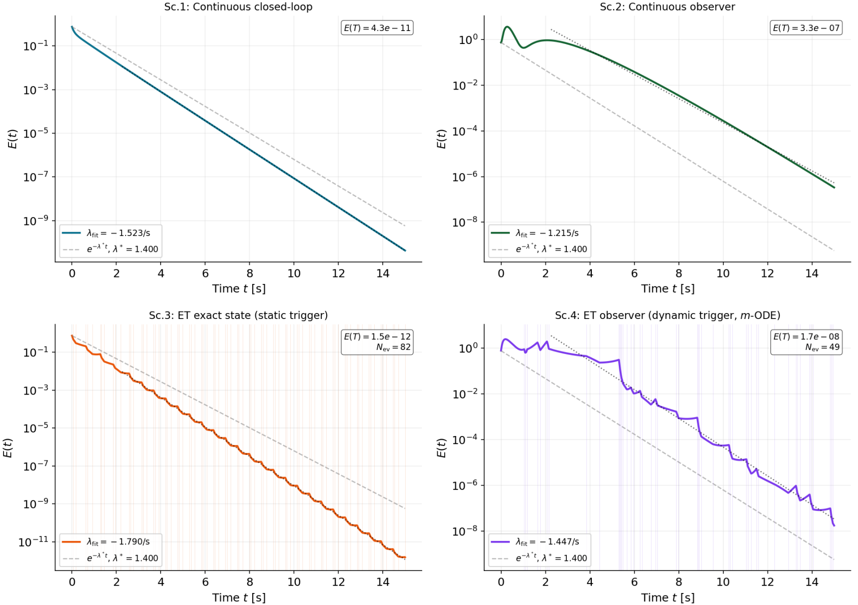
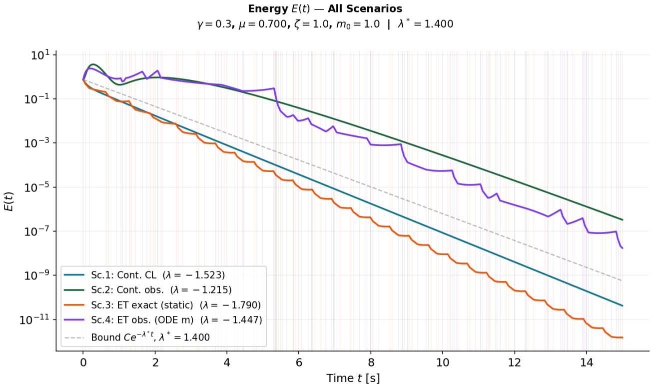
$$A = \begin{pmatrix} 0.3 & 0.6 & 0.4 \\ 0.6 & -1.0 & 0.6 \\ 0.4 & 0.6 & 0.3 \end{pmatrix}, \quad B = e_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad C = e_3^\top = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix} \tag{29}$$

Parameters

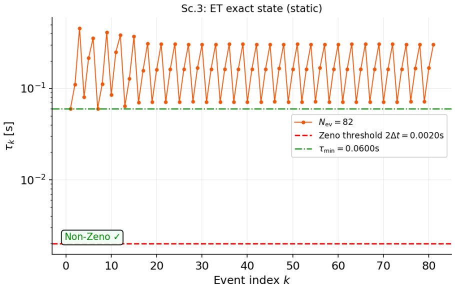
Description	Symbol	Value
Design rule	γ	0.30
$m(\cdot)$ rate	μ	0.70
Coupling	ζ	1.0
Initial state	z_0	$(1, 0.5, -0.5)$
Initial observer	$\hat{z}_0$	$(0, 0, 0)$
Initial internal	m_0	1.0
Time horizon	T	$15\,\mathrm{s}$
Time step	Δt	$10^{-3}\,\mathrm{s}$

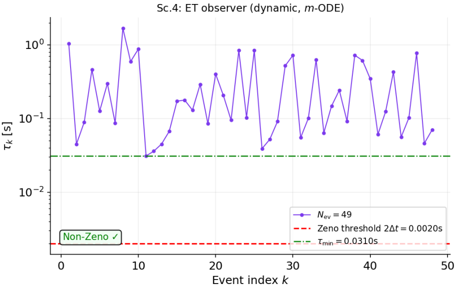
ET Observer Control in $\mathbb{R}^3$ — Energy $E(t)$
Gains: Riccati (LQR) | $\gamma = 0.3$, $\mu = 0.700$, $\zeta = 1.0$, $m_0 = 1.0$ | $\lambda^* = 1.400$

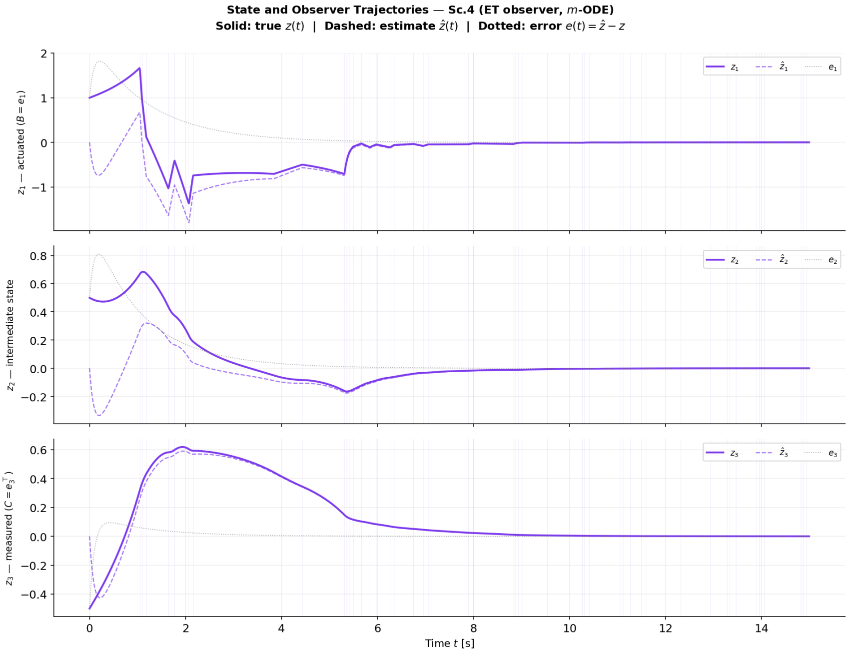


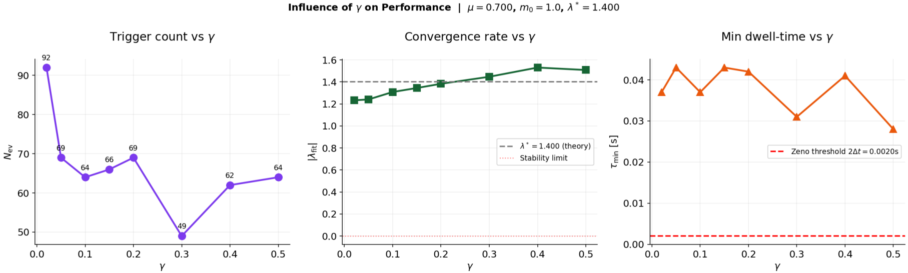

Energy $E(t)$ — All Scenarios
$\gamma = 0.3$, $\mu = 0.700$, $\zeta = 1.0$, $m_0 = 1.0$ | $\lambda^* = 1.400$
$E(t)$
Time t [s]
Sc.1: Cont. CL ($\lambda = -1.523$)
Sc.2: Cont. obs. ($\lambda = -1.215$)
Sc.3: ET exact (static) ($\lambda = -1.790$)
Sc.4: ET obs. (ODE m) ($\lambda = -1.447$)
Bound $Ce^{-\lambda^* t}$, $\lambda^* = 1.400$

Inter-Event Intervals $\tau_k = t_{k+1} - t_k$ — No-Zeno Verification






State and Observer Trajectories — Sc.4 (ET observer, m-ODE)
Solid: true z(t) | Dashed: estimate ẑ(t) | Dotted: error e(t) = ẑ − z
z1 — actuated (B = e1)
z2 — intermediate state
z3 — measured (C = e3ᵀ)
Time t [s]
z1
ẑ1
e1
z2
ẑ2
e2
z3
ẑ3
e3


Influence of γ on Performance | μ = 0.700, m0 = 1.0, λ* = 1.400
Trigger count vs γ
Nev
92
69
64
66
69
49
62
64
γ
Convergence rate vs γ
|λfit|
λ* = 1.400 (theory)
Stability limit
γ
Min dwell-time vs γ
τmin [s]
Zeno threshold 2Δt = 0.0020s
γ

PDE model on $(0,1)$

$$\partial_{tt} z = c^2 \partial_{xx} z - d\partial_t z + b(x)u(t_k) \tag{30}$$

1st-order form $\mathbf{z} = (u, v)^\top$, $v = z_t$

$$\dot{\mathbf{z}} = \underbrace{\begin{pmatrix} 0 & I \\ c^2\partial_{xx} & -dI \end{pmatrix}}_{\mathcal{A}} \mathbf{z} + \mathcal{B}u(t_k)$$

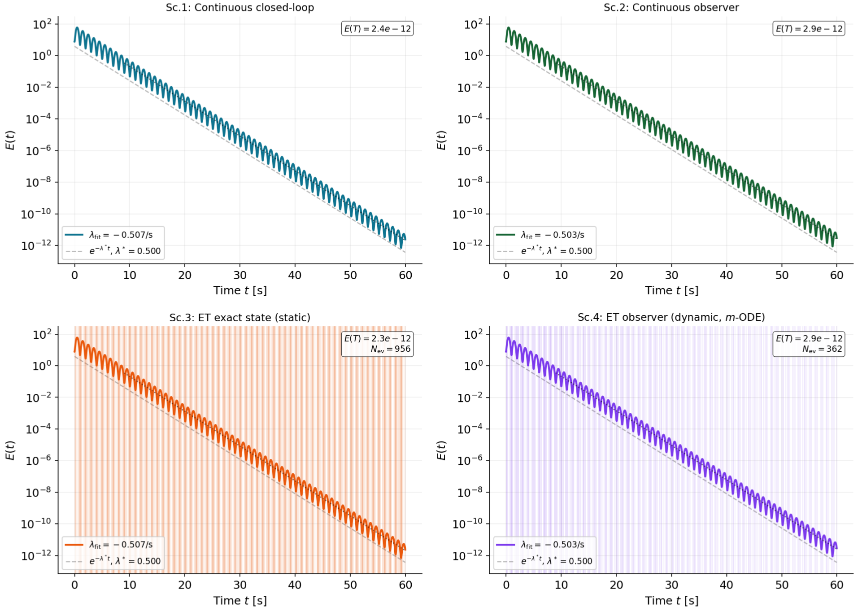
Actuation/observation

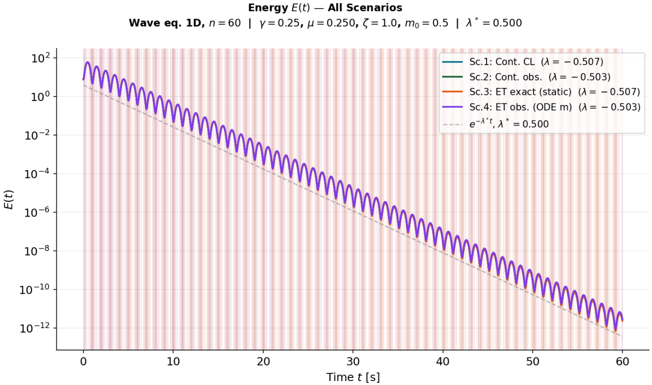
Force on v at $x_a \approx 0$ and measure u at $x_o \approx 1$.

Parameters

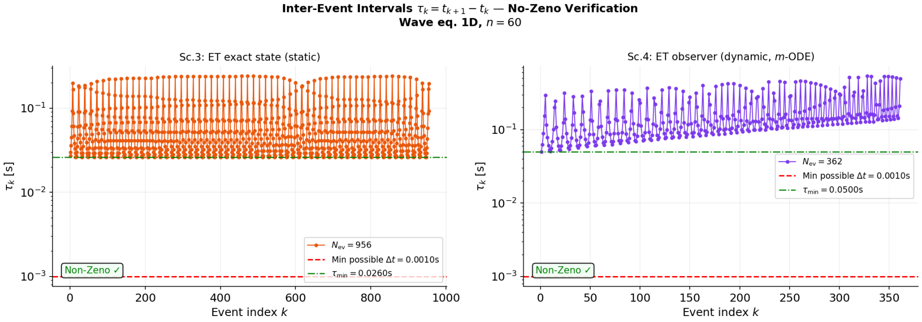
Description	Symbol	Value
Wave speed	c	1.0
Damping	d	0.5
Actuation	x_a	≈ 0
Observation	x_o	≈ 1
Design rule	γ	0.25
$m(\cdot)$ rate	μ	0.25
Initial internal	m_0	0.5
Time horizon	T	$60\,\mathrm{s}$
Time step	Δt	$10^{-3}\,\mathrm{s}$

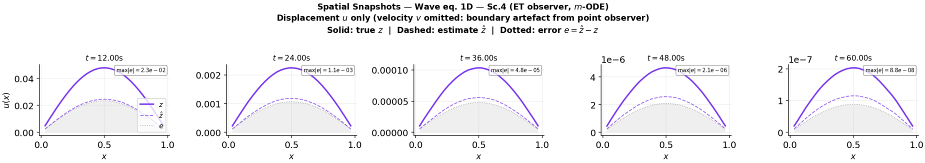
ET Observer Control — Wave eq. 1D, $n = 60$ — Energy $E(t)$
Gains: Riccati (LQR) | $\gamma = 0.25$, $\mu = 0.250$, $\zeta = 1.0$, $m_0 = 0.5$ | $\lambda^* = 0.500$

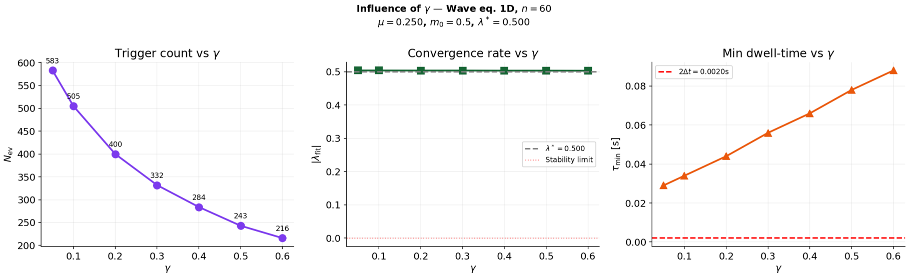




Energy $E(t)$ — All Scenarios
Wave eq. 1D, $n = 60$ | $\gamma = 0.25$, $\mu = 0.250$, $\zeta = 1.0$, $m_0 = 0.5$ | $\lambda^* = 0.500$
Sc.1: Cont. CL ($\lambda = -0.507$)
Sc.2: Cont. obs. ($\lambda = -0.503$)
Sc.3: ET exact (static) ($\lambda = -0.507$)
Sc.4: ET obs. (ODE m) ($\lambda = -0.503$)
$e^{-\lambda^* t}$, $\lambda^* = 0.500$
$E(t)$
Time t [s]


Inter-Event Intervals $\tau_k = t_{k+1} - t_k$ — No-Zeno Verification
Wave eq. 1D, $n = 60$
Sc.3: ET exact state (static)
τ_k [s]
10^{-1}
10^{-2}
10^{-3}
Non-Zeno ✓
$N_{ev} = 956$
Min possible $\Delta t = 0.0010$s
$\tau_{min} = 0.0260$s
0
200
400
600
800
1000
Event index k
Sc.4: ET observer (dynamic, m-ODE)
τ_k [s]
10^{-1}
10^{-2}
10^{-3}
$N_{ev} = 362$
Min possible $\Delta t = 0.0010$s
$\tau_{min} = 0.0500$s
Non-Zeno ✓
0
50
100
150
200
250
300
350
Event index k


Spatial Snapshots — Wave eq. 1D — Sc.4 (ET observer, m-ODE)
Displacement u only (velocity v omitted: boundary artefact from point observer)
Solid: true z | Dashed: estimate ẑ | Dotted: error e = ẑ − z
t = 12.00s
max|e| = 2.3e − 02
t = 24.00s
max|e| = 1.1e − 03
t = 36.00s
max|e| = 4.8e − 05
t = 48.00s
max|e| = 2.1e − 06
t = 60.00s
max|e| = 8.8e − 08
u(x)
x


Influence of γ — Wave eq. 1D, n = 60
μ = 0.250, m0 = 0.5, λ* = 0.500
Trigger count vs γ
Nev
583
505
400
332
284
243
216
γ
Convergence rate vs γ
|λfit|
λ* = 0.500
Stability limit
γ
Min dwell-time vs γ
τmin [s]
2Δt = 0.0020s
γ

Reaction-diffusion 2D

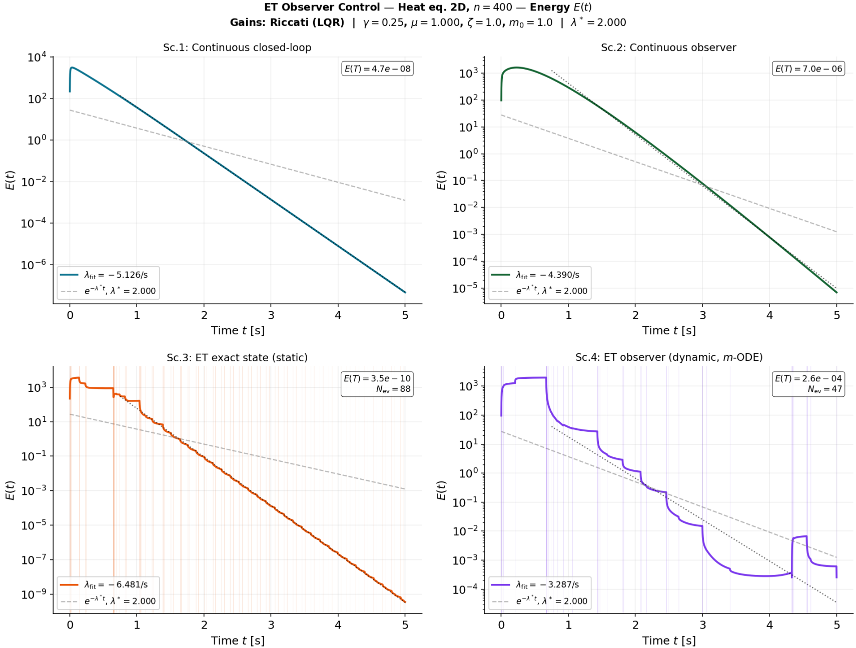
PDE model on $\Omega = (0,1)^2$

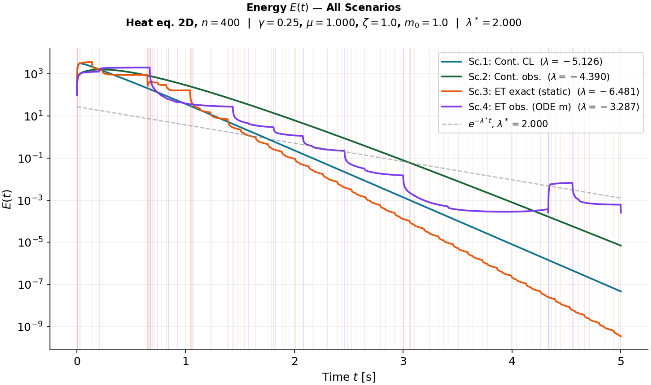
$$\partial_t z = \underbrace{\kappa \Delta z + \sigma z}_{\mathcal{A}} + b(\mathbf{x}) u(t_k) \tag{31}$$

$\max \mathrm{Re}(\mathcal{A}) \approx +1.58$ *(unstable)*

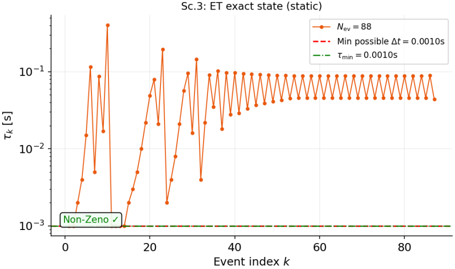
Parameters

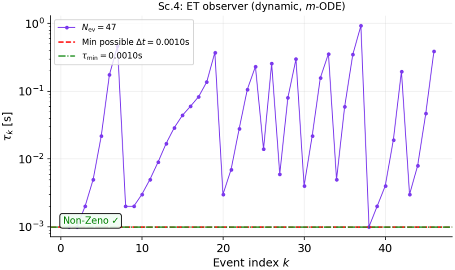
Description	Symbol	Value
Diffusivity	κ	0.25
Reaction	σ	7.50
Mesh	$n_1 \times n_2$	20×20
Actuation	$\mathbf{x}_a$	corner $(0,0)$
Observation	$\mathbf{x}_o$	corner $(1,1)$
Design rule	γ	0.20
$m(\cdot)$ rate	μ	1.0
Initial internal	m_0	1.0
Time horizon	T	$5\,\mathrm{s}$
Time step	Δt	$10^{-3}\,\mathrm{s}$


ET Observer Control — Heat eq. 2D, n = 400 — Energy E(t)
Gains: Riccati (LQR) | γ = 0.25, μ = 1.000, ζ = 1.0, m0 = 1.0 | λ* = 2.000
Sc.1: Continuous closed-loop
E(T) = 4.7e − 08
λfit = −5.126/s
e^{−λ*t}, λ* = 2.000
Sc.2: Continuous observer
E(T) = 7.0e − 06
λfit = −4.390/s
e^{−λ*t}, λ* = 2.000
Sc.3: ET exact state (static)
E(T) = 3.5e − 10
Nev = 88
λfit = −6.481/s
e^{−λ*t}, λ* = 2.000
Sc.4: ET observer (dynamic, m-ODE)
E(T) = 2.6e − 04
Nev = 47
λfit = −3.287/s
e^{−λ*t}, λ* = 2.000
E(t)
Time t [s]

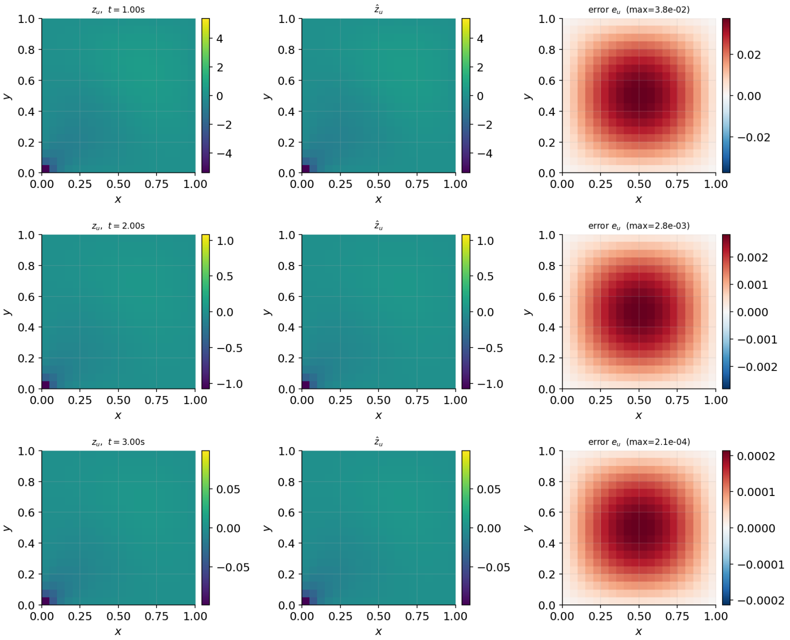

Energy $E(t)$ — All Scenarios
Heat eq. 2D, $n = 400$ | $\gamma = 0.25$, $\mu = 1.000$, $\zeta = 1.0$, $m_0 = 1.0$ | $\lambda^* = 2.000$
Sc.1: Cont. CL ($\lambda = -5.126$)
Sc.2: Cont. obs. ($\lambda = -4.390$)
Sc.3: ET exact (static) ($\lambda = -6.481$)
Sc.4: ET obs. (ODE m) ($\lambda = -3.287$)
$e^{-\lambda^* t}$, $\lambda^* = 2.000$
$E(t)$
Time t [s]

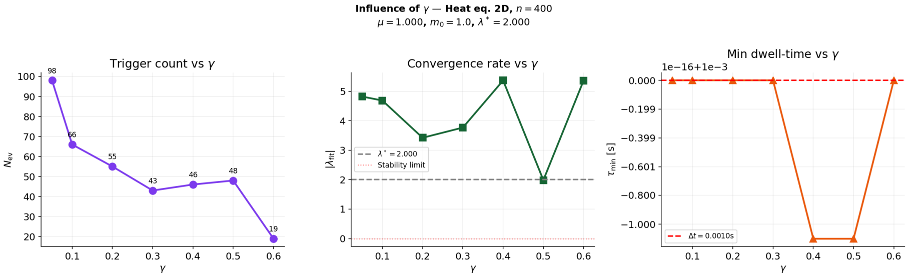
Inter-Event Intervals $\tau_k = t_{k+1} - t_k$ — No-Zeno Verification
Heat eq. 2D, $n = 400$





Spatial Snapshots — Heat eq. 2D — Sc.4 (ET observer, m-ODE)
Col 1: true z | Col 2: estimate $\hat{z}$ | Col 3: error e




Influence of γ — Heat eq. 2D, n = 400
μ = 1.000, m0 = 1.0, λ* = 2.000
Trigger count vs γ
N_ev
98
66
55
43
46
48
19
γ
Convergence rate vs γ
|λ_fit|
λ* = 2.000
Stability limit
γ
Min dwell-time vs γ
1e−16+1e−3
τ_min [s]
0.000
−0.199
−0.399
−0.601
−0.800
−1.000
Δt = 0.0010s
γ

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- Observer-based event-triggered control of infinite-dimensional systems
  $\Rightarrow$ well-posedness, exclusion of Zeno and exponential stability established
- Systems governed by (dynamics) operators **not required to be skew-adjoint**
  $\Rightarrow$ we rely on **quasi-dissipativity** property
- Role of dynamic triggering in observer-based PDE setting fundamental $\Rightarrow$ *static triggering rules may be insufficient to guarantee absence of Zeno behaviour*

What has been achieved

- **Finite dimension**: uniform dwell-time $\tau_{\min} > 0$ independent of k proved from

$$\left(\gamma \|\hat{x}(t_{k+1})\|_{\mathcal{H}} + m(t_{k+1})\right)^2 \leq 2(t_{k+1} - t_k) \left\|\gamma \|\hat{x}(\cdot)\|_{\mathcal{H}} + m(\cdot)\right\|_{L^\infty(t_k, t_{k+1})} \left\|\dot{\hat{x}}\right\|_{L^\infty(t_k, t_{k+1}; \mathcal{H})}, \tag{32}$$

- **Infinite dimension + spectral decomposition**: uniform dwell-time when **unstable subspace $\mathcal{H}_u$ is finite-dimensional** (parabolic systems)

Open problem: general infinite dimension

When A generates a C_0-semigroup with **no spectral gap** bound on $\|\dot{\hat{z}}(t_k)\|$ grows like $e^{ct_k} \to +\infty$: **only No-Zeno is currently provable, not a uniform dwell-time**

Perspectives - Relaxing quasi-dissipativity via a P-norm

What has been achieved

Assumption 19 requires a quasi-dissipativity property in $\mathcal{H}$-norm can be weakened
$\Rightarrow$ it exists a Lyapunov quadratic functional $V : \mathcal{H} \to \mathbb{R}_+$ associated to $A + BK$ such that

$$\exists m_1, m_2 > 0, \quad m_1 \|z\|^2_{\mathcal{H}} \leq V(z) \leq m_2 \|z\|^2_{\mathcal{H}}, \quad \forall z \in \mathcal{H}, \tag{33}$$

$$\exists P \in \mathcal{L}(\mathcal{H}), \quad P = P^*, \qquad V(z) = \|Pz\|^2_{\mathcal{H}}, \quad \forall z \in \mathcal{H}, \tag{34}$$

$$\exists \beta > 0, \quad \forall z_0 \in D(A), \quad \frac{d}{dt} V\left(e^{t(A+BK)} z_0\right) \leq -2\beta V\left(e^{t(A+BK)} z_0\right), \quad \forall t \geq 0, \tag{35}$$

Open directions

- **Non-self-adjoint** P: multiplier-based Lyapunov functionals for hyperbolic systems framework allows $P \neq P^*$, but *constructing Gramian in this case requires new admissibility results*
- **Boundary observation** (C_{obs} unbounded): extend Gramian framework to *Salamon-Weiss well-posed systems class*

Two distinct sampling problems

So far: control $u(t) = K\hat{z}(t_k)$ is sampled, observation $y(t) = Cz(t)$ is **continuous**

Two natural extensions:

1. **Triggered observations, continuous control**: y is only transmitted at instants $\{\tau_j\}$, control $u(t) = K\hat{z}(t)$ is updated continuously using observer
2. **Both triggered**: control sampled at $\{t_k\}$, observation transmitted at $\{\tau_j\}$, two independent or coupled trigger rules

What changes

- Observer error $e(\cdot)$ no longer satisfies a simple ODE: between two observation instants, $\dot{e} = (A + LC)e$ but L acts on a stale output $y(\tau_j)$, not on $y(t)$

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Yi, Xiaodong, Karl H. Johansson, and Dimos V. Dimarogonas (2017). "Dynamic Event-Triggered and Self-Triggered Control for Multi-Agent Systems". In: IEEE Transactions on Automatic Control 62.7, pp. 3307–3319.

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Well-posedness

Theorem (Well-posedness of a maximal solution)

Under Assumptions (G) and Assumption (M), there exist a strictly increasing sequence $(t_k)_{k\in\mathbb{N}} \subset \mathbb{R}_+$ *with* $t_0 = 0$ *and a* **unique mild solution**

$$Z \in C([0, T^*); \mathcal{H} \times \mathcal{H}), \tag{36}$$

where $T^* = \sup_k t_k \in (0, \infty]$, *such that*

$$Z(t) = \exp((t - t_k)\mathcal{A}_{\mathrm{aug}})Z(t_k) + \int_{t_k}^{t} \exp((t-s)\mathcal{A}_{\mathrm{aug}}) \begin{pmatrix} BK\hat{z}(t_k) \\ 0 \end{pmatrix} ds, \quad \forall t \in [t_k, t_{k+1}), \tag{37}$$

and triggering rule (17) holds on each interval $[t_k, t_{k+1})$ *for* $k \in \mathbb{N}$,

Key points

- *Existence and uniqueness* despite **implicit coupling** (t_{k+1} depends on the solution, which itself depends on t_{k+1})
- Sequence $(t_k)_{k\in\mathbb{N}}$ is constructed *simultaneously* with solution by induction (no a priori knowledge of trigger times is needed)

Regularity of solutions

Theorem (Piecewise classical regularity)

Under Assumptions (G) *and Assumption* (M), *and if in addition* $Z_0 \in \mathcal{D}(A) \times \mathcal{D}(A)$ *then* $Z(\cdot)$ *is* **weakly continuous** *in* $[0, T^*)$ *with values in* $\mathcal{D}(A) \times \mathcal{D}(A)$ *and* **piecewise classical** *in* $[0, T^*)$ *with values in* $\mathcal{H} \times \mathcal{H}$, *i.e.*

$$Z \in C_w((t_k, t_{k+1}); \mathcal{D}(A) \times \mathcal{D}(A)) \cap C^1((t_k, t_{k+1}); \mathcal{H} \times \mathcal{H}), \quad \forall k \in \mathbb{N}. \tag{38}$$

Moreover, it holds that for all $t \in [0, T^*)$,

$$\left\|\dot{\hat{z}}\right\|_{L^\infty(0,t;\mathcal{H})} \leq e^{(\|BK\|+c_A)t} \left\|(A+BK)\hat{z}_0 + LCe_0\right\|_{\mathcal{H}} + te^{(\|BK\|+c_A)t} \left\|LC\right\| M_0 \left\|(A+LC)e_0\right\|_{\mathcal{H}}, \tag{39}$$

Key points

- Regularity is *piecewise*: at each trigger time t_k, control $u = K\hat{z}(t_k)$ is frozen creating a jump in $\dot{\hat{z}}(\cdot)$ but not in $\hat{z}(\cdot)$ itself
- Bound on $\|\dot{\hat{z}}\|_{L^\infty}$ grows in t: this is *not uniform* in k for general infinite-dimensional A and is **key obstruction to a uniform dwell-time without additional structure**

Theorem (No zeno)

Under Assumptions (G) *and Assumption* (M), *solution to system* (**??**) *subjected to triggering rule* (17) *satisfies*

$$T^* = \infty, \tag{40}$$

and thus does **not experience any Zeno phenomenon**

Key points

- Zeno behaviour (infinitely many triggers in finite time) would make system *physically unrealisable*
- Dynamic variable $m(\cdot) > 0$ is *essential ingredient*: it **prevents trigger threshold from collapsing to zero** even as $\hat{z}(t) \to 0$
- Without $m(\cdot)$, Zeno *can* occur for classical static triggers as $\hat{z} \to 0$

Exponential stability

Theorem (Event-triggered stability)

Choosing small enough γ, under Assumptions (G)-(M), then event-triggered system (12)-(17) is **exponentially stable** *in $\mathcal{H} \times \mathcal{H}$ for an initial datum in $\mathcal{D}(A) \times \mathcal{D}(A)$, i.e.*

$$\exists \hat{C} > 0, \quad \exists \hat{\lambda} > 0, \quad \|\hat{z}(t)\|_{\mathcal{H}}^2 + \|e(t)\|_{\mathcal{H}}^2 \leq \hat{C} \left(\|\hat{z}_0\|_{\mathcal{H}} + \|e_0\|_{\mathcal{H}} + m_0 \right) e^{-\hat{\lambda} t}, \quad \forall t \geq 0, \tag{41}$$

Key points

- Rate $\hat{\lambda} = \min(2\beta_1, 2\mu, 2\alpha_0)$ balances three competing decays: closed-loop, dynamic variable $m(\cdot)$ and observer error
- Condition on γ is **explicit**:
  larger $\gamma \Rightarrow$ fewer triggers $\Rightarrow$ smaller $\beta_1 \Rightarrow$ slower convergence
  $\Rightarrow$ *trade-off* between communication rate and stability rate
- Term m_0 in bound (41) captures initial trigger threshold:
  $\Rightarrow$ setting $m_0 \to 0$ recovers continuous-time rate

Assumption (Riesz spectral decomposition)

Operator $A\colon \mathcal{D}(A) \subset \mathcal{H} \to \mathcal{H}$ is a Riesz-spectral operator with discrete spectrum

$$\sigma(A) = \{\lambda_n\}_{n\geq 1}, \quad \lim_{n\to\infty} \mathrm{Re}(\lambda_n) = -\infty,$$

Fix a threshold $\lambda_c > 0$ and define

$$\sigma_u := \{\lambda \in \sigma(A), \quad \mathrm{Re}(\lambda) \geq -\lambda_c\}, \quad N := |\sigma_u| < \infty, \quad \sigma_s := \sigma(A) \setminus \sigma_u, \tag{42}$$

Let $P_u, P_s \in \mathcal{L}(\mathcal{H})$ be corresponding Riesz spectral projectors satisfying

$$P_u + P_s = I, \quad P_u P_s = P_s P_u = 0,$$

$$\mathcal{H}_u := P_u \mathcal{H} \cong \mathbb{C}^N, \quad (\textit{finite-dimensional}), \quad \mathcal{H}_s := P_s \mathcal{H}, \quad (\textit{infinite-dimensional}), \tag{43}$$

$$A_u := A|_{\mathcal{H}_u} \in \mathcal{L}(\mathcal{H}_u),$$

and generator of an (analytic) exponentially stable semigroup on $\mathcal{H}_s$,

$$A_s := A|_{\mathcal{D}(A)\cap\mathcal{H}_s},$$

satisfying

$$\exists M_s \geq 1, \quad \exists \alpha_s > \lambda_c > 0, \quad \|\exp(tA_s)\|_{\mathcal{L}(\mathcal{H}_s)} \leq M_s e^{-\alpha_s t}, \qquad t \geq 0, \tag{44}$$