Hierarchical dynamic domain decomposition methods for the multiscale Boltzmann equation

Domenico CAPARELLO, Department of Mathematics and Computer Science - Ferrara
Lorenzo PARESCHI, Department of Mathematics and Computer Science - Ferrara
Thomas REY, Laboratoire J.A. Dieudonné - Nice
Tommaso TENNA, Department of Mathematics 'Guido Castelnuovo' - Roma

We present a hierarchical dynamic domain decomposition method for the Boltzmann equation [1, 2] based on moment realizability matrices, a concept introduced by Levermore, Morokoff, and Nadiga. This criterion is used to dynamically and automatically partition the two-dimensional spatial domain into two regimes : the Euler regime, and the kinetic Boltzmann regime. The key advantage of this approach lies in the use of Euler equations in regions where the flow is near hydrodynamic equilibrium, and the Boltzmann equation where strong non-equilibrium effects dominate, such as near shocks and boundary layers. This allows for both high accuracy and significant computational savings, as the Euler solver is considerably cheaper than the kinetic Boltzmann model.
We propose an extension of this approach to a multi-level hierarchical dynamic domain decomposition method, in which the spatial domain is divided into more than two regimes.
A novel coupling strategy for Euler and kinetic regimes is proposed, ensuring the preservation of high order accuracy across the hybrid interface. The strategy is validated through a computational framework implementing state-of-the-art numerical techniques. This combination enables robust and scalable simulations of multi-scale kinetic flows with complex geometries, effectively bridging the gap between computational efficiency and physical fidelity.

[1] D. Caparello, L. Pareschi, T. Rey. *Hierarchical dynamic domain decomposition for the multiscale boltzmann equation*, 2025. doi :10.48550/arXiv.2505.03360.

[2] D. Caparello, T. Tenna. *A coupled imex domain decomposition method for high-order time integration of the es-bgk model of the boltzmann equation*, 2025. doi :10.48550/arXiv.2512.03586.

Contact : `domenico.caparello@unife.it`