

## Fully spectral method for the linearized Boltzmann BGK equation

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In this talk I will present a spectral scheme for the linearized Boltzmann equation which preserves several invariants as well as the long time asymptotic. The equation is a collisional model, posed on the whole space, in which particles are confined by a potential  $\phi$ . It is known since Boltzmann that the space of steady states (i.e. solutions of the equation lying in the kernel of the collision operator) depends strongly on the potential [2]. It contains in general the constants and the hamiltonian, but if  $\phi$  is harmonic, then it also contains additional time-periodic modes. Each of these mode is paired with a time invariant quantity (mass, energy...). It has been shown in [1] that the solution converges at exponential rate toward a unique linear combination of these mode. This result was obtained by using hypocoercivity techniques : it consists in finding a suitable norm, in which the sum of the transport and collision operators is coercive.

After an introductory example of hypocoercivity for matrices, we will focus the scheme. It is obtained by projecting the equation on the basis of Hermite polynomials in velocity, and on orthonormal polynomials with respect to the weight  $e^{-\phi}$  in space, thus defining a fully spectral scheme [3]. We will show that it preserves every invariants as well as hypocoercivity. Since our choice of basis is not classic, we will discuss the computation of the orthonormal polynomials sequence for the weight  $e^{-\phi}$ . Finally, simulations will be presented.

- [1] K. Carrapatoso, J. Dolbeault, F. Hérau, S. Mischler, C. Mouhot, C. Schmeiser. *Special macroscopic modes and hypocoercivity*. Journal of the European Mathematical Society, 2023. 65 pages, 1 figure.
- [2] C. Cercignani. *The Boltzmann Equation and Its Applications*. Applied Mathematical Sciences. Springer New York, 2012.
- [3] B. Grosse. *Fully spectral scheme for the linear BGK equation on the whole space*, 2025. Working paper or preprint.