

# Spectral analysis and global dynamics of a structured SIS model with mutation

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We study a trait-structured SIS epidemiological model with mutation. This framework describes a heterogeneous infected population and leads to an infinite-dimensional nonlinear dynamical system in  $L^1(\Omega)$ .

Due to the global nonlinear coupling, the system does not generate a monotone dynamical system, which prevents the use of standard comparison techniques.

Our approach is therefore based on the spectral analysis of a family of positive operators arising from the linearization around the disease-free equilibrium. In this setting, the spectral bound  $s(L_0)$  plays the role of a generalized reproduction number.

A central aspect of the analysis is the role of the minimal recovery rate

$$\underline{\gamma} := \inf_{x \in \Omega} \gamma(x).$$

When  $\underline{\gamma} > 0$ , we obtain a complete and sharp threshold description : the sign of  $s(L_0)$  characterizes extinction, critical behavior, and persistence, together with the existence of a unique endemic equilibrium.

In contrast, when  $\underline{\gamma} = 0$ , this characterization no longer holds. However, we show that the condition  $s(L_0) > 0$  still ensures persistence of the infection. The structure of endemic equilibria and the full long-time dynamics in this singular regime remain more intricate and only partially understood.

Finally, in a simplified framework, we study the stability of endemic equilibria and show their robustness with respect to small perturbations of the recovery rate and the transmission parameters.

## References

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