A spatio-temporal random synthetic turbulent velocity field: The underlying Gaussian structure and its Lagrangian properties

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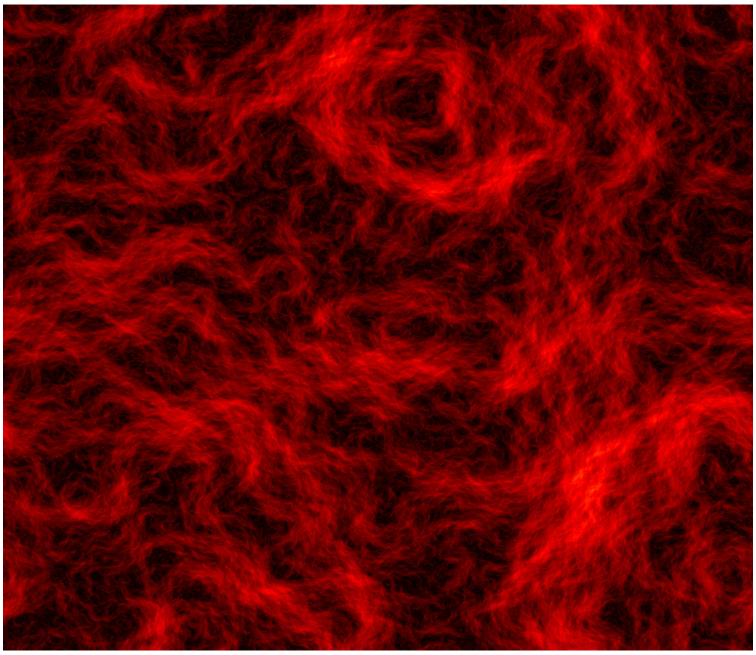
We develop in [1], simulate and extend an initial proposition by Chaves et al. [2] concerning a random incompressible vector field able to reproduce key ingredients of three-dimensional turbulence in both space and time. In this work we focus on the important underlying Gaussian framework. Presently, the statistical spatial structure of this velocity field is consistent with a divergence-free fractional Gaussian vector field that encodes all known properties of homogeneous and isotropic fluid turbulence at a given finite Reynolds number, up to second-order statistics. The temporal structure of the velocity field is introduced through a stochastic evolution of the respective Fourier modes. In the simplest picture, Fourier modes evolve according to an Ornstein–Uhlenbeck process, where the characteristic time scale depends on the wave-vector amplitude. For consistency with direct numerical simulations (DNS) of the Navier–Stokes equations, this time scale is inversely proportional to the wave-vector amplitude. As a consequence, the characteristic velocity that governs the eddies is independent of their size and is related to the velocity standard deviation, which is consistent with some features of the so-called sweeping effect. To ensure differentiability in time while respecting the Markovian nature of the evolution, we use the methodology developed by Viggiano et al. [3] to propose a fully consistent stochastic picture. Both exact predictions and numerical estimations of the model are compared with DNS provided by the Johns Hopkins database [4]. Finally, we numerically integrate the flow equation $\dot{\boldsymbol{x}} = \boldsymbol{u}(t, \boldsymbol{x}(t))$ to obtain the trajectories of tracers advected by the Eulerian fields and analyse their statistical behaviour [5]. Specifically, we examine a set of trajectories to characterise the properties of single-particle motion by determining their acceleration and velocity increment statistics. We eventually compare these results to experimental data on velocity and acceleration correlation functions [6].



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[1] Chatelain et al., *Journal of Fluid Mechanics*, (2026), **1030**, A23.
[2] Chaves et al., *J. Stat. Phys.*, (2003), **113(5-6)**, 643-692.
[3] Viggiano et al., *Journal of Fluid Mechanics*, (2020), **900**, A27.
[4] Li et al., *J. Turbulence*, (2008), **9**, 31.
[5] Reneuve et al., *Physical Review Letters*, (2020), **125(1)**, 014502.
[6] Sawford et al., *Physics of fluids*, (2003), **15**, 3478-3489.