CANUM2026 01-05 juin 2026

Modeling constrained Cosserat beams for control problems

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This presentation deals with modelling and analysis of a system of nonlinear PDE called the Cosserat equations, which describe the dynamics of deformable beams.

The field of soft robotics has been significantly developing over the last few decades. Soft robots, which are made of continuous flexible materials in contrast to rigid robots, offer a promising new point of view for complex tasks in interaction with changing environments, such as locomotion. This point of view takes inspiration from the locomotion abilities of animals. In particular, many soft locomotors have elongated shapes, either in biological (snakes, eels) or robotic systems [2].

Deriving dynamical equations for elongated flexible materials (called beams) is a long-standing mechanical modelling problem. In particular, large deformations can be handled with the formalism of Cosserat equations [3], stated about a century ago. However, because they are a system of partial differential equations (PDEs), open questions remain about their well-posedness, on which there are very few studies, often in restrictive cases [1]. Moreover, having in mind the applications in robotics, it is suitable to study these equations as an infinite-dimensional control problem and to include important mechanical constraints within the Cosserat equations.

Therefore, in this talk, I will present the derivation of the constrained Cosserat equations and discuss its well-posedness and controllability.

First, using a Lagrangian formalism on the a matrix Lie group G and Euler-Poincaré reduction, I will obtain a variational problem of the Lie algebra $\mathfrak{g}$. Based on duality and functional operators, this variational problem is a unified formalism that allows to take into account any mechanical constraint on the beam (for example, inextensibility, or obstacles in the environment). While an important distinction is usually made between holonomic and nonholonomic constraints, this formalism allows to treat both constraint types simultaneously. Then, I will emphasize the unconstrained Cosserat equations that can be written as a system of nonlinear hyperbolic partial differential equations in the following way :

$$\begin{cases} I\partial_t\eta - H\partial_x(\xi-\xi_0) - \mathrm{ad}^*_\eta I\eta + \mathrm{ad}^*_\xi H(\xi-\xi_0) = 0 \\ \partial_x\eta - \partial_t\xi = [\eta,\xi] \\ \xi(t,0) = \xi_0(0), \quad \xi(t,1) = \xi_0(1) \\ (\eta(0,x),\xi(0,x)) = (\eta^{in}(x),\xi^{in}(x)) \end{cases} \tag{1}$$

where $\eta,\xi : [0,T]\times[0,1] \to \mathfrak{g}$ are the unknowns, $\mathrm{ad}_\eta\,\xi = [\eta,\xi]$ is the Lie bracket generated by the Lie group, $H, I \in \mathscr{L}(\mathfrak{g},\mathfrak{g})$ and $\eta^{in}, \xi^{in}, \xi_0 : [0,1] \to \mathfrak{g}$. Finally, I will briefly present the future directions for the study of the controllability of snake-inspired robots, modeling forces as control, and give some preliminary results on the well posedness (local existence in time in $\mathscr{C}^k([0,T]\times[0,1];\mathfrak{g}^2)$) and the controllability.

[1] S. S. Antman, T. I. Seidman. *The parabolic-hyperbolic system governing the spatial motion of nonlinearly viscoelastic rods.* Archive for rational mechanics and analysis, **175(1)**, 85–150, 2005.

[2] C. Armanini, F. Boyer, A. T. Mathew, C. Duriez, F. Renda. *Soft robots modeling : A structured overview.* IEEE Transactions on Robotics, **39(3)**, 1728–1748, 2023.

[3] E. M. P. Cosserat, F. Cosserat. *Théorie des corps déformables.* A. Hermann et fils, 1909.

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