Numerical simulation of the three dimensional Vlasov-Poisson system in a torus

Francis FILBET, IMT - Université de Toulouse
L. Miguel RODRIGUES, IRMAR - Université de Rennes
Kim Han TRINH, IRMAR - Université de Rennes

We propose and study a Particle-In-Cell (PIC) method utilizing modified Crank-Nicolson time discretization for tokamak equilibrium configurations in torus geometries. These configurations are described by the three-dimensional Vlasov-Poisson system with a strong, inhomogeneous external magnetic field. In this regime, traditional explicit schemes are constrained by stability conditions linked to the small Larmor radius and plasma frequency [3, 2]. To avoid this limitation, our approach is based on numerical schemes [1, 2], providing a consistent PIC discretization of the asymptotic system taking into account variations of the magnetic field. We carry out some theoretical proofs and perform several numerical experiments to validate the method, demonstrating its robustness and accuracy, especially in the complex geometries of a torus.

[1] F. Filbet, L. Rodrigues, K. Trinh. *Convergence analysis of a Crank-Nicolson scheme for strongly magnetized plasma*, 2025.

[2] F. Filbet, L. Rodrigues, K. Trinh. *A modified Crank-Nicolson scheme for the Vlasov-Poisson system with a strong external magnetic field.* Journal of Computational Physics, **552**, 2026.

[3] L. Ricketson, L. Chacón. *An energy-conserving and asymptotic-preserving charged-particle orbit implicit time integrator for arbitrary electromagnetic fields.* Journal of Computational Physics, **418**, 2020.

Contact : trinh.kim-han@univ-rennes.fr