Un modèle réduit exact pour la conception d'un jumeau numérique d'un réseau de distribution d'eau basé sur l'assimilation de données en temps réel

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Overview

- Introduction (CoRREau project & PhD thesis)
- Reduced-Order Model
- Digital Twin
- Conclusion & perspectives
- Articles & communications

CoRREau

CoRREau project
Resilience by Design and Security of Water Networks

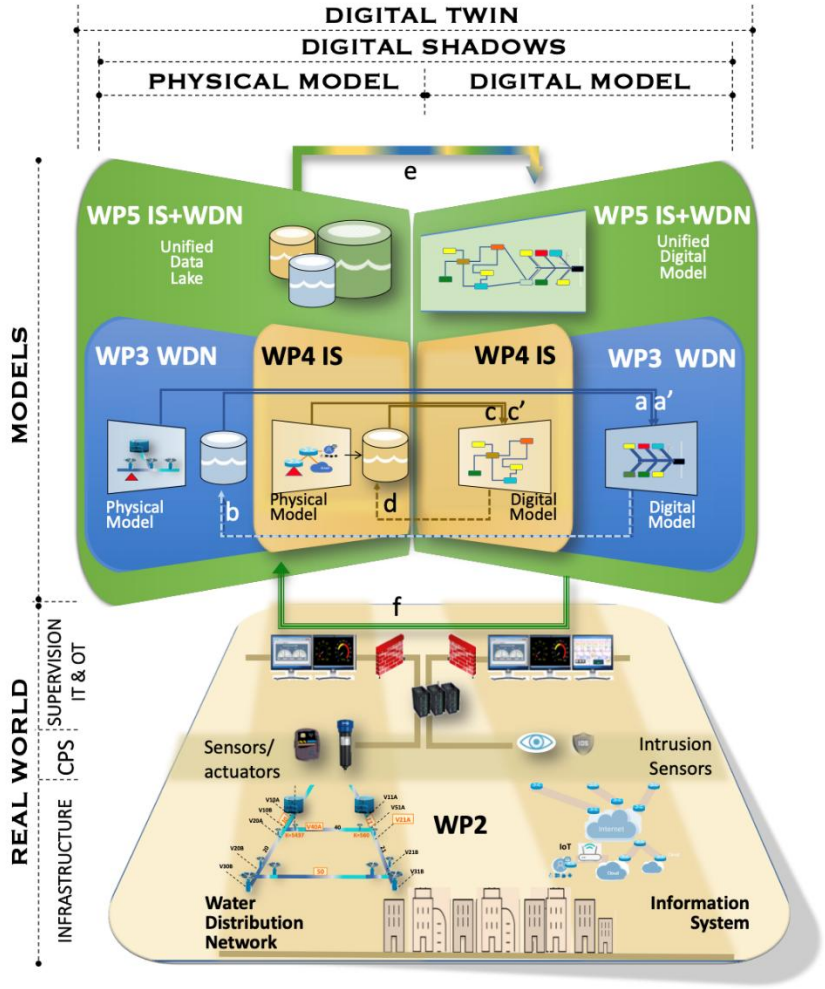
The **CoRREau** project integrates a "**digital twin**" using real-time data to model the WDN and a "**digital shadow**" trained and then activated on the basis of historical data from the computer network.

↓

Complete digital twin

↓

Case studies will be based on scenarios of interest **proposed by the water department of Eurométropole Strasbourg in France** (CUS).



PhD thesis

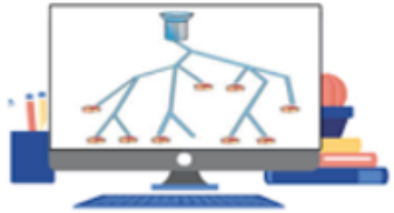
Towards a digital twin for resilient and sustainable adaptation of water networks

CoRREau

Complex network

Data

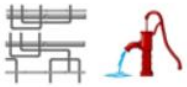
Action on the physical system:
controlling, monitoring and decision-making



Reduced order model

Diversity of Components

Pipelines ...



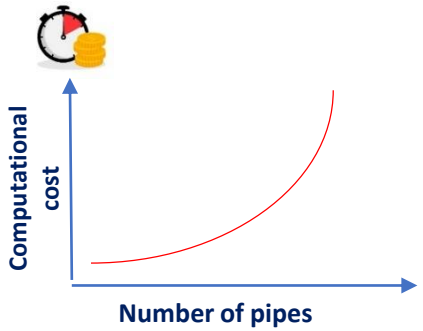
Interconnection

Geographical Span

> 50000 pipes

$$\begin{cases} \boldsymbol{\xi}(\mathbf{q}) - \mathbf{A}^T\mathbf{h} - \mathbf{A}_r^T\mathbf{h}^R = \mathbf{0}_{n_p} \\ -\mathbf{A}\mathbf{q} - \mathbf{d} = \mathbf{0}_{n_j} \end{cases}$$

System of linear & non-linear equations



The objectives:

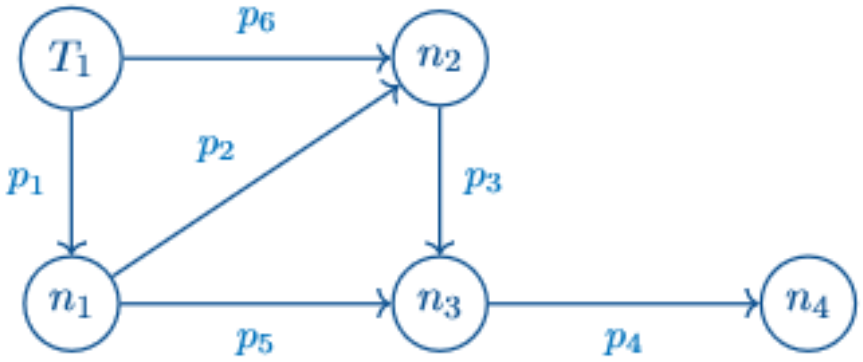
- to propose an order reduction of the system
- to develop a **digital twin** based on the assimilation of real-time observation data.
- To quantify and study the propagation of uncertainty

RÉPUBLIQUE FRANÇAISE
INRAE
ETTIS
le cnam cedric EA4629
Strasbourg.eu eurométropole
ICUBE
le cnam M2N Modélisation mathématique et numérique

1-Reduced Order Model

Mathematical modeling/Tools

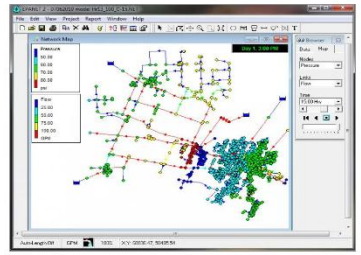
$$\begin{cases} \boldsymbol{\xi}(\mathbf{q}) - \mathbf{A}^T\mathbf{h} - \mathbf{A}_r^T\mathbf{h}^R & = \mathbf{0}_{n_p} \\ -\mathbf{A}\mathbf{q} - \mathbf{d} & = \mathbf{0}_{n_j} \end{cases}$$



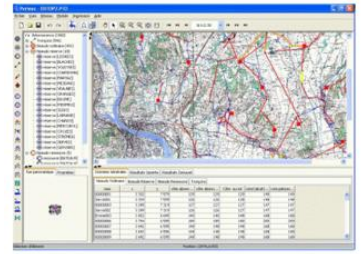
$$A^N = \begin{array}{c} \\ n_1 \\ n_2 \\ n_3 \\ n_4 \\ T_1 \end{array} \begin{array}{c} \begin{array}{cccccc} p_1 & p_2 & p_3 & p_4 & p_5 & p_6 \end{array} \\ \begin{pmatrix} -1 & 1 & 0 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 & 0 & -1 \\ 0 & 0 & -1 & 1 & -1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \end{array}$$

Graph-Based Representation of Water Networks

EPANET



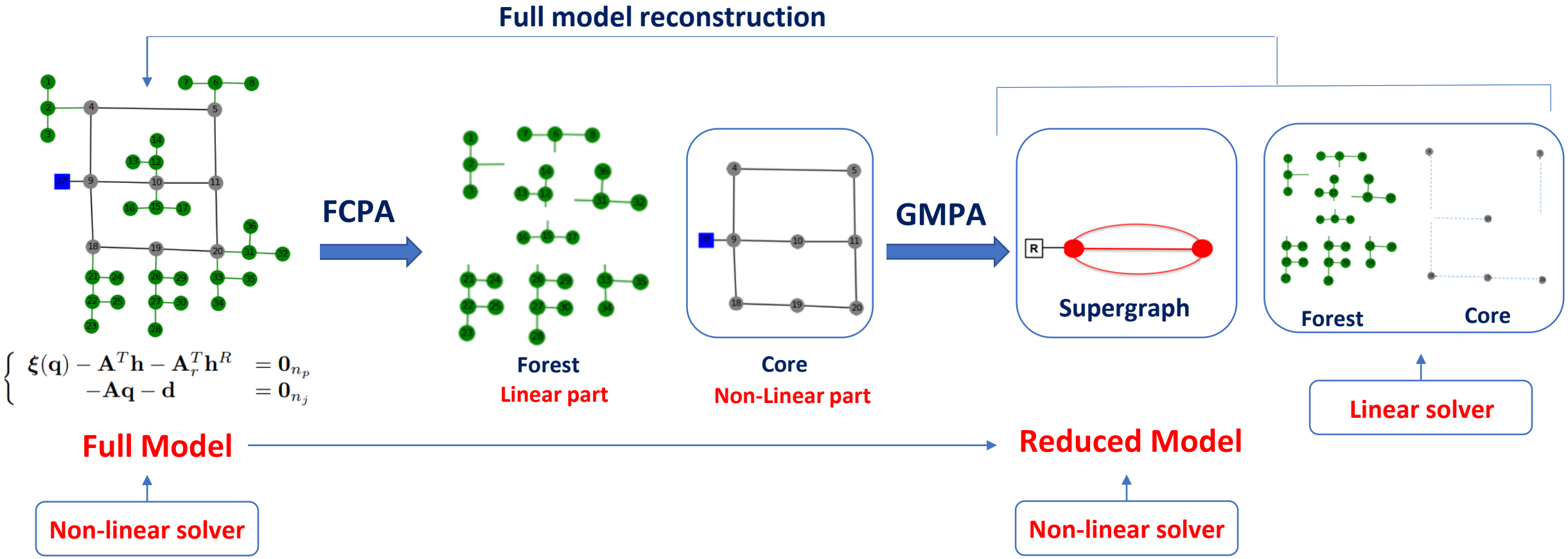
Porteau



Hydraulic simulation software

1-Reduced Order Model

Forest-Core Partitioning Algorithm (FCPA)[1] / Graph Matrix Partitioning Algorithm (GMPA)[2]

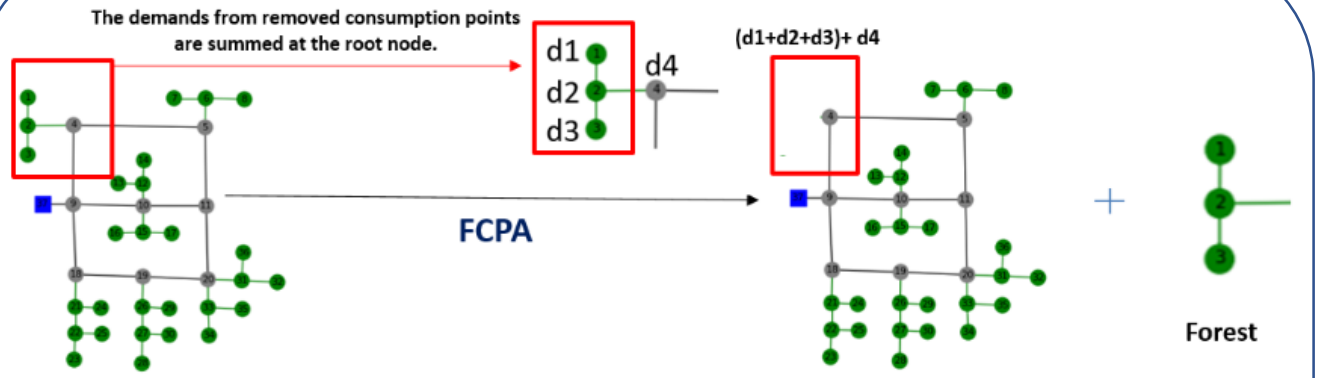


[1]Simpson, A.R.: Elhay, S.; Alexander, B. Forest-Core Partitioning Algorithm for Speeding up Analysis of Water Distribution Systems. J. Water Resour. Plann. Manag. 2014. 140, 435–443

[2]Deuerlein, J.; Elhay, S.; Simpson, A.R. Fast Graph Matrix Partitioning Algorithm for Solving the Water Distribution System Equations. J. Water Resour. Plann. Manag. 2016, 142, 04015037

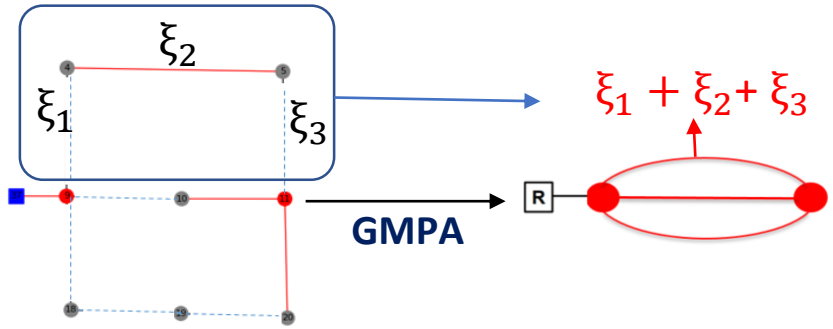
1-Reduced Order Model

No loss of information



Summation at the root node of the demands providing from the removed consumption points in the FCPA procedure.

$$\mathbf{d} = \mathbf{d}^C - \mathbf{A}_{CF}\mathbf{A}_{FF}{}^{-1}\mathbf{d}^F$$



Summation of head losses on the paths between supernodes.

$$\begin{cases} \mathbf{M}_0\bar{\boldsymbol{\xi}}(\bar{\mathbf{q}}) - \mathbf{M}_0\mathbf{A_s}^T\bar{\mathbf{h}} - \mathbf{M}_0\bar{\mathbf{e}} = \mathbf{0}_{n_l+n_s} \\ -\mathbf{A}_s\bar{\mathbf{q}} - \mathbf{d}^s = \mathbf{0}_{n_s}. \end{cases}$$

1-Reduced Order Model

Application of FCPA & GMPA to WDN

EPANET file input

Computing the flow and head of **links and nodes** in **the forest**

linear solver

Calculation of flow and head for **intermediate nodes and links** in **the core**

Linear solver

Computing the flow of **superlinks** and **supernode**'s head In the subgraph

Non-linear solver

OOPNET

Read

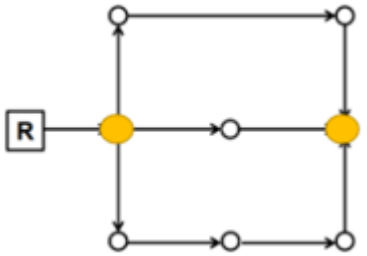
NetworkX

OOPNET Network Object

which can be manipulated in Python syntax.

Incidence matrix

FCPA



Core

GMPA

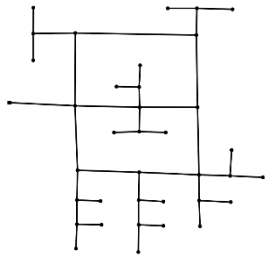
Prim

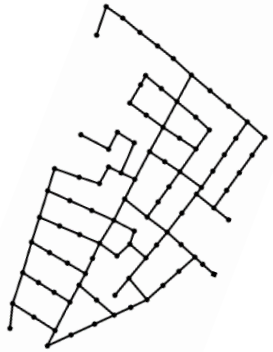
Subgraph

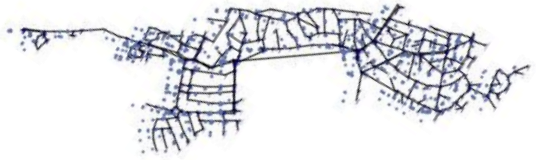
1-Reduced Order Model

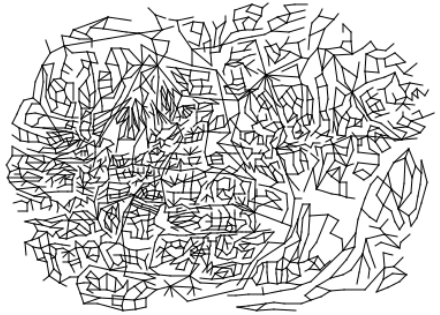
Networks

In what follows, tests are carried out on different networks in order to validate the FCPA and GMPA, the Prim algorithm and the hydraulic calculation (supergraph, core and forest).









Networks

Issue 1

Full network: 934 pipes, 856 nodes, 8 reservoir

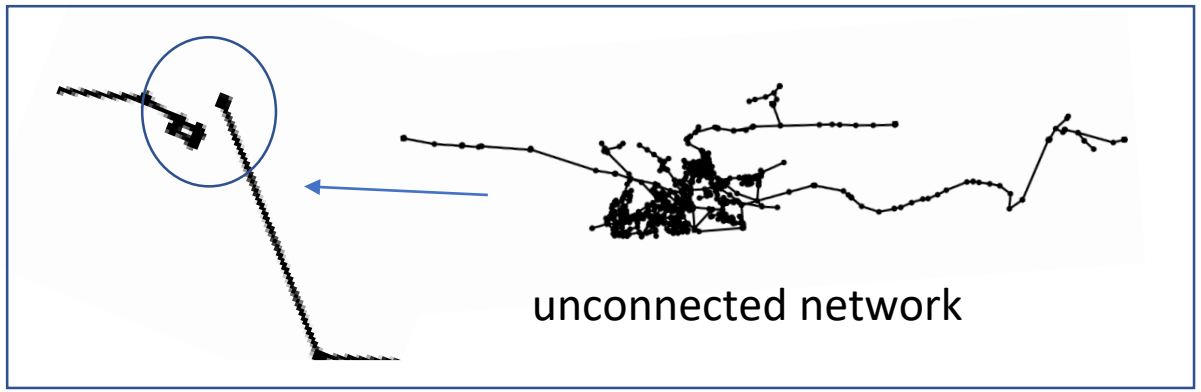
→ **FCPA** →

Forest: 361 pipes, 361 nodes — **Core**: 573 pipes, 487 nodes

→ **GMPA** →

Subgraph: **? superlinks**, 160 supernodes

Prim's algorithm does not work



Networks

Update the Prim algorithm: detect connected parts and apply Prim to each connected component, starting with the reservoir.

Full network: 934 pipes, 856 nodes, 8 reservoir

→ **FCPA**

Forest: 361 pipes, 361 nodes

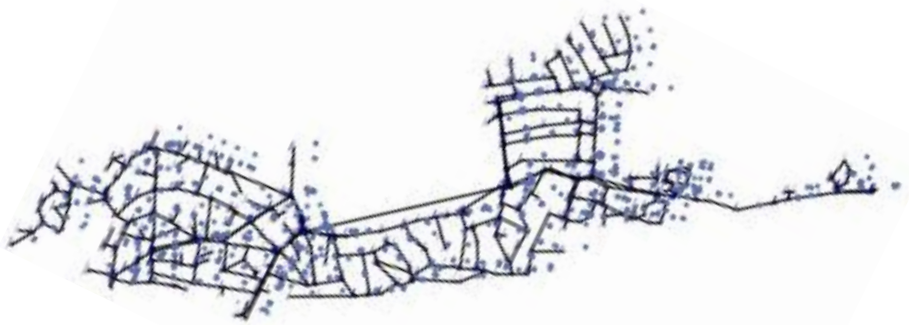
Core: 573 pipes, 487 nodes

→ **GMPA**

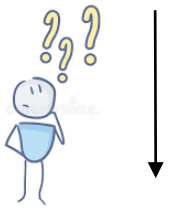
Subgraph: 246 superlinks, 160 supernodes

Networks

Issue 2

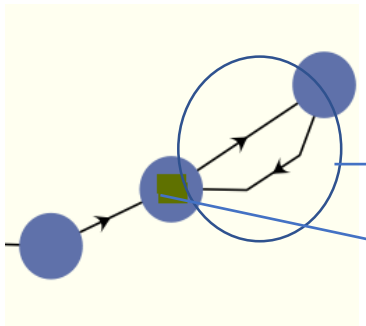


Full network { 1118 pipes; 1039 nodes; 2 reservoir } → **FCPA** → **Forest** { ? pipes; ? nodes } **Core** { ? pipes; ? nodes } → **GMPA** → **Subgraph** { ? superlinks; ? supernodes }



Warning !

Parallel pipes can be merged silently

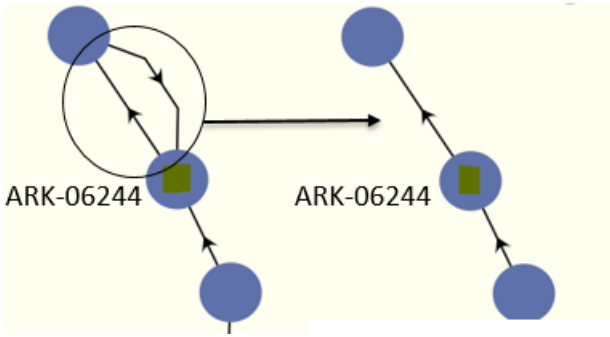


Presence of parallel pipes

Artificial supernode

Networks

Update the code: to detect the presence of parallel arcs and replace them with equivalent arcs.



Full network: 1118 pipes, 1039 nodes, 2 reservoir → **1115 pipes, 1039 nodes, 2 reservoir**

detect parallel arcs and replace them with equivalent arcs

FCPA

Forest: 330 pipes, 330 nodes; **Core**: 785 pipes, 709 nodes

GMPA

Subgraph: 226 superlinks, 150 supernodes

1-Reduced Order Model

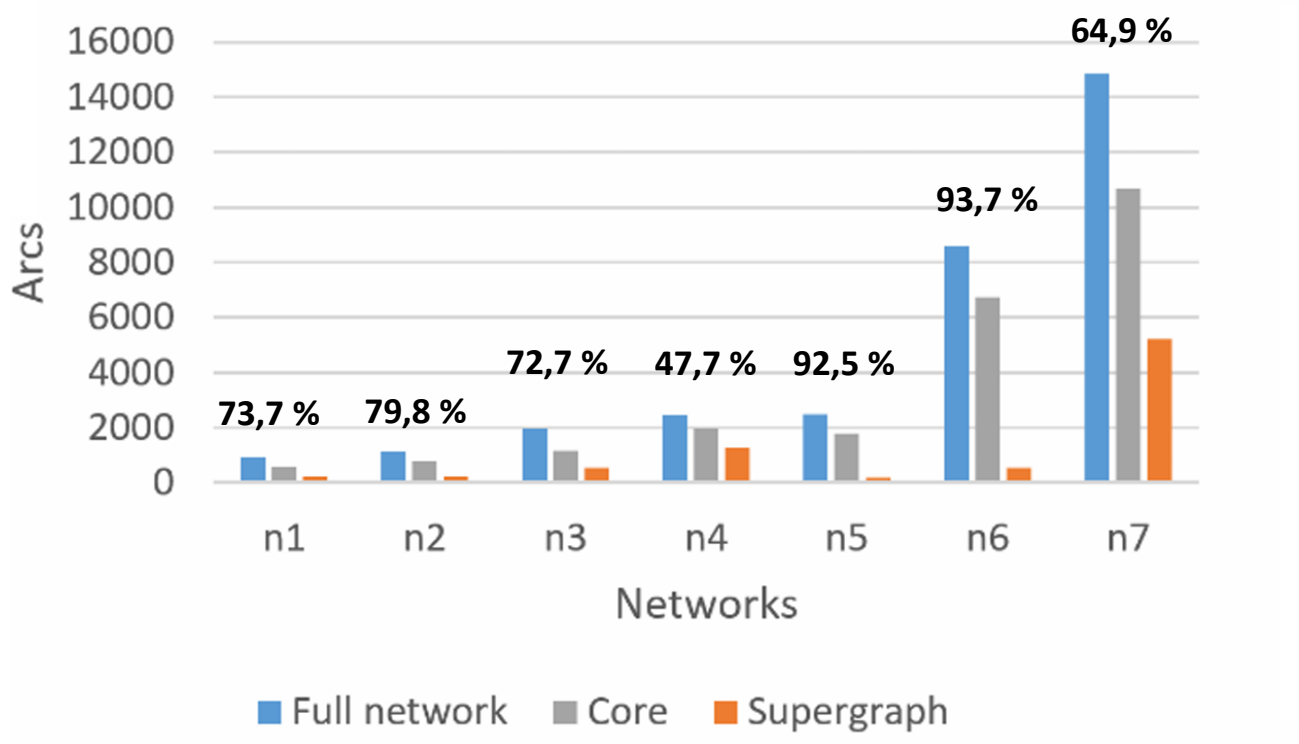
Networks

WDNs	Arcs	Junct. nodes	Sources	Equiv. arcs	ncc
n1	934	848	8	0	6
n2	1118	1039	2	3	1
n3	1976	1770	4	3	2
n4	2465	1890	1	49	1
n5	2510	2445	2	0	1
n6	8584	8392	2	1	1
n7	14830	12523	7	508	2

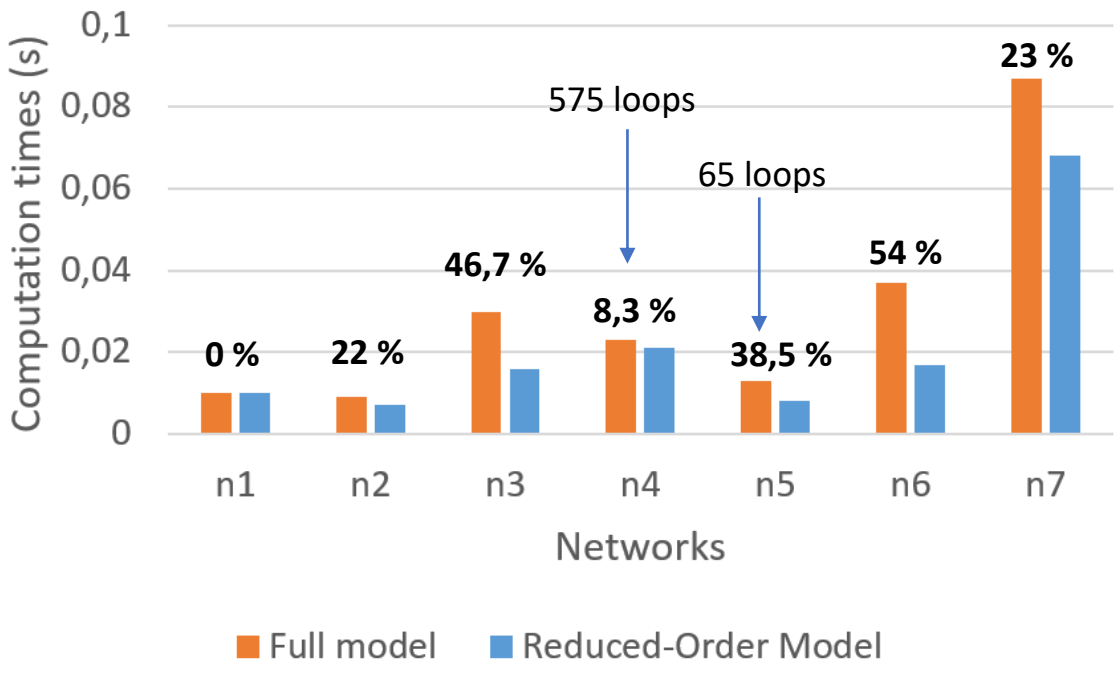
Table 01: Characteristics of the 7 networks considered for validation. For each of the 7 networks, the number of arcs, nodes, sources, equivalent arcs (after replacing parallel arcs), and the number of connected components (ncc) is mentioned.

1-Reduced Order Model

Results: size & calculation time reduction



- The progressive reduction in size of the nonlinear system.

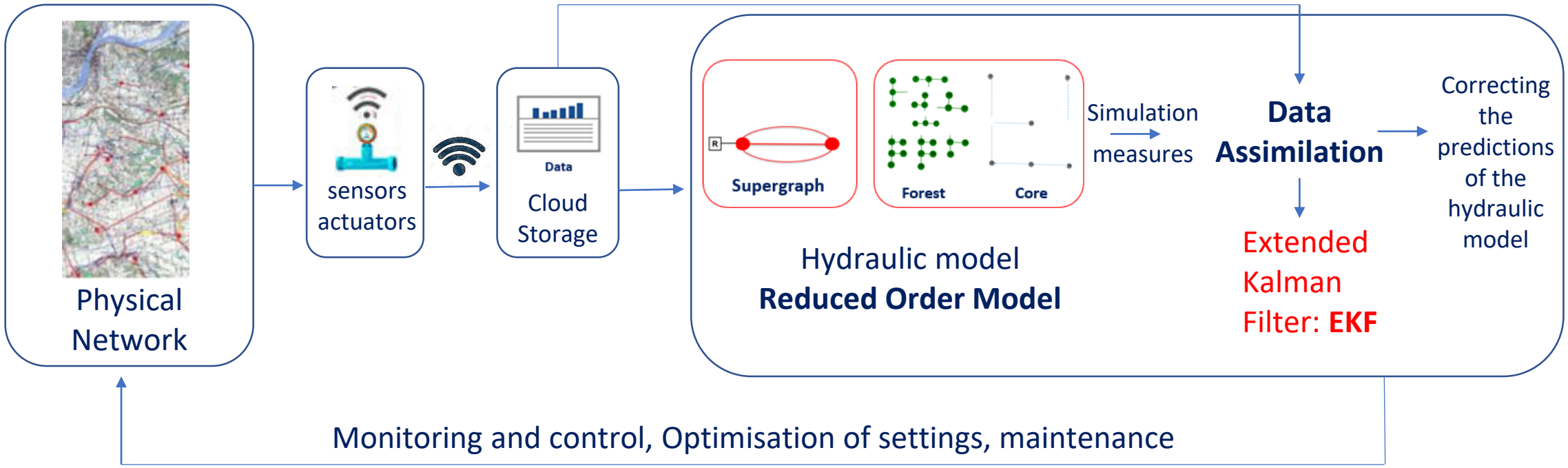


- Computation times of the full and reduced model for the different networks

Digital Twin

Physical system

Virtual representation



2- Digital Twin

$$\begin{cases} \boldsymbol{\xi}(\mathbf{q}) - \mathbf{A}^T\mathbf{h} - \mathbf{A}_r^T\mathbf{h}_r = \mathbf{0}_{n_a} \\ -\mathbf{A}\mathbf{q} - \mathbf{d} = \mathbf{0}_{n_j} \end{cases}$$

$$\frac{dh_r}{dt} = -S_r^{-1}A_r(A^{-1}d)$$

Kalman Filter

Update the hydraulic model

1-Initialisation

The initial level of the tanks

2-Prediction

Predicting tank levels at time t+1

3-Observation

Define the measurement model (which may depend on tank levels, flow rate, etc.)

4-Correction

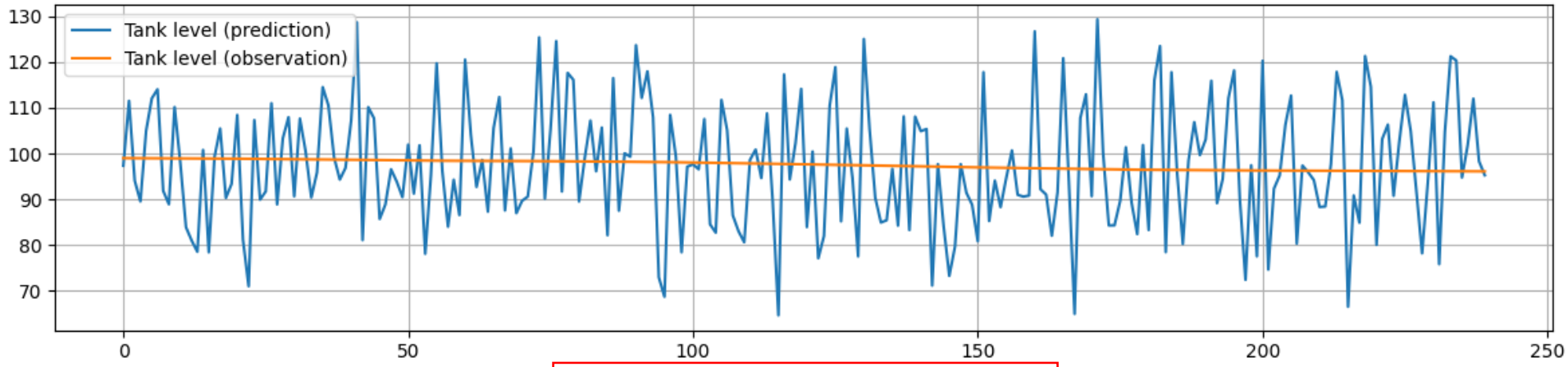
Correct tank levels using measurements and Kalman gain

Data Assimilation - Kalman filter -

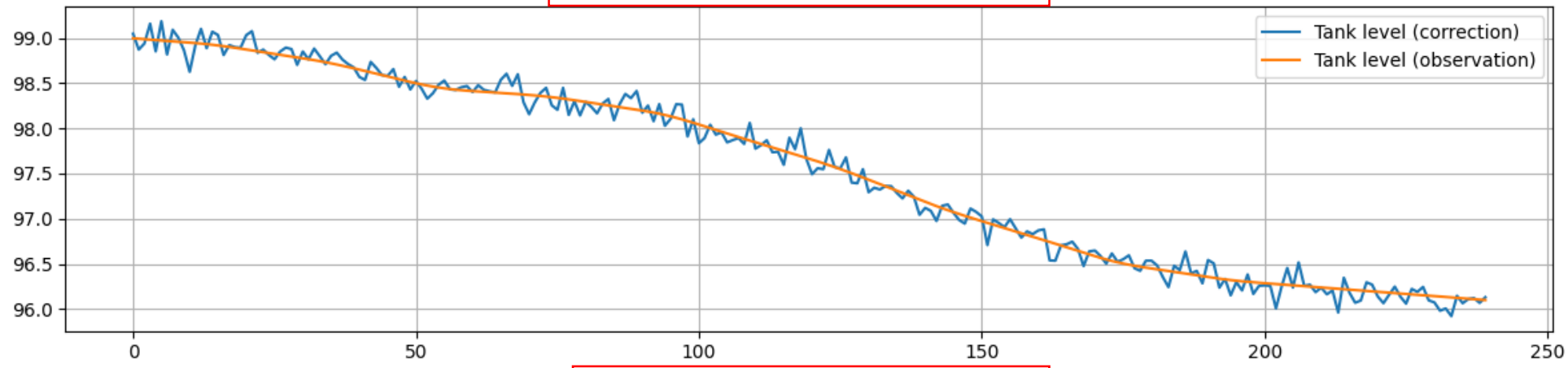
2- Digital Twin

Results

Tank level



Error | MAE = 11.0028 | RMSE = 13.5005



Error | MAE = 0.0819 | RMSE = 0.1046

$\omega_k \in [-2, 2,8]$ m

$\vartheta_k \in [-0,3, 0,25]\ m$

$Time\ step = 6\ min$

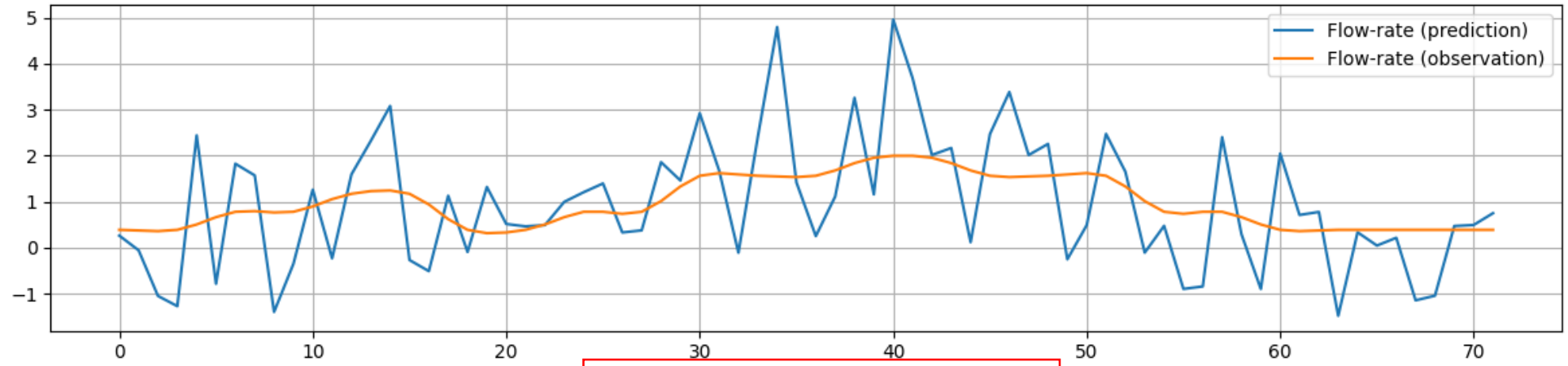
ω_k: *Process noise*

ϑ_k: *Measurement noise*

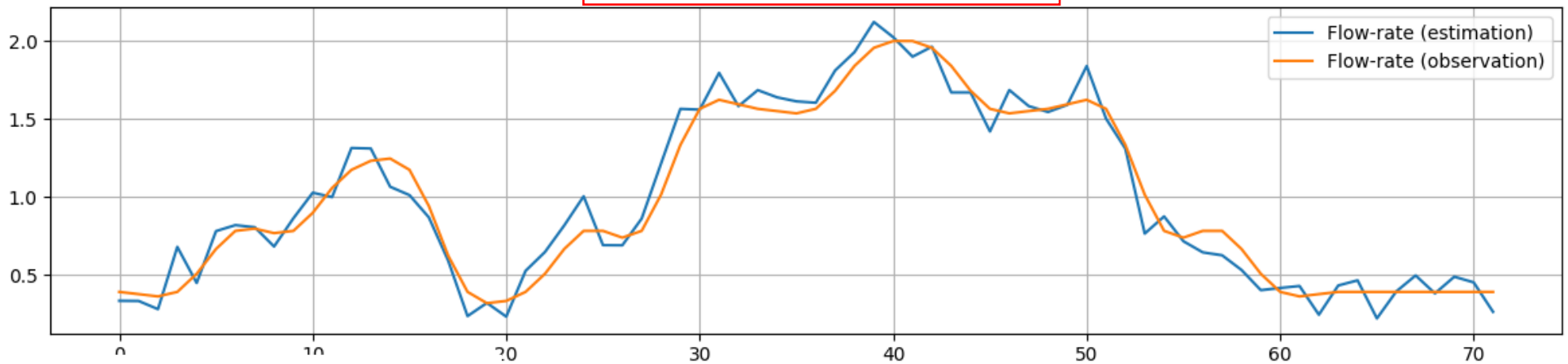
2- Digital Twin

Results

Flow-rate



Error | MAE = 0.9448 | RMSE = 1.1811



Error | MAE = 0.0965 | RMSE = 0.1169

$\omega_k \in [-33, 32]\ l/s$

$\vartheta_k \in [-0{,}3, 0{,}3]\ l/s$

$Time\ step = 20\ min$

ω_k: *Process noise*

ϑ_k: *Measurement noise*

Conclusion & perspectives

- The linear Kalman filter has been validated and demonstrated satisfactory results by reducing the errors between predictions and corrections, compared to the measured data.
- The next step would be to implement the Extended Kalman Filter (EKF) in order to handle the nonlinearity of the system.
- This work leads to a complete digital twin integrating real-time data and advanced modeling for accurate, efficient, and resilient management of the water distribution network.

Articles & communications

- **Cheima Djemel**, Olivier Piller, Thierry Horsin, Chloé Mimeau, Iraj Mortazavi. **Review of Reduced-Order Models for Online Protection of Water Distribution Networks**. Engineering Proceedings, 2024, 69 (1), pp.159. ⟨10.3390/engproc2024069159⟩. ⟨hal-04570712⟩.
- **Cheima Djemel**, Olivier Piller, Thierry Horsin, Chloé Mimeau, Iraj Mortazavi. **Reduced-order Model of Water Distribution System Based on Graph Decomposition for Digital Twin Design**. 2025. ⟨hal-05250427⟩. [preprint]
- **Cheima Djemel**, Olivier Piller, Thierry Horsin, Chloé Mimeau, Iraj Mortazavi. **Reduced-Order Model Using Graph Decomposition for Water Distribution Networks**. Journal of Hydroinformatics. 2026 [submitted]

- **The 3rd International Joint Conference on Water Distribution Systems Analysis & Computing and Control for the Water Industry (WDSA/CCWI 2024).** Ferrara, Italy. July 2024.
- **SimHydro 2025: Which data for water and models?** Nice, France. June 2025.
- **4th International Joint Conference on Water Distribution Systems Analysis and Computing and Control in the Water Industry (WDSA/CCWI 2026).** Paphos, Cyprus. May 2026. (Abstract accepted)
- **47th National Congress on Numerical Analysis**, June 1–5 in Saint-Jacut-de-la-Mer, France.
- **XVIII International Congress of Civil Engineering and the 53rd National Meeting of FEMCIC,** August 26 to 29, 2026, at the *Centro Educativo y Cultural Gómez Morin,* Querétaro. [*Keynote Speaker]*

Thank you

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