A Comparative Assessment of Moment-Based Closures for Flows at Various Levels of Rarefaction

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Accurate simulation of flows with spatially varying rarefaction levels poses significant challenges to conventional numerical methods. Stochastic approaches such as Direct Simulation Monte Carlo (DSMC) become computationally prohibitive in dense flow regions, while Navier-Stokes-based CFD solvers lose validity where continuum assumptions break down. Hybrid CFD-DSMC methods have been developed to address this issue [9, 7], yet remain sensitive to the placement of the coupling interface [10, 3].
The method of moments offers a promising unified alternative, generating a hierarchy of macroscopic conservation equations from the Boltzmann equation that naturally incorporates non-equilibrium effects. Truncation of this hierarchy introduces an underdetermined system, requiring a closure relation for the unknown fluxes. We present a systematic comparison of closure strategies, including Grad-type [5, 6], maximum entropy [8, 1], and quadrature-based approaches [2, 4], assessed against reference Discrete Velocity Method (DVM) solutions over a wide range of rarefaction levels with BGK collisions. Special attention is devoted to the structural properties of each closure, namely realizability and hyperbolicity, and their influence on solution accuracy. The results provide practical guidelines for selecting appropriate moment closures based on flow conditions and computational requirements.

[1] M. R. A. Abdelmalik, E. H. van Brummelen. *Moment closure approximations of the boltzmann equation based on φ-divergences.* Journal of Statistical Physics, **164**, 77–104, 2016.

[2] C. Chalons, R. Fox, M. Massot. *A multi-gaussian quadrature method of moments for gas-particle flows in a LES framework.* In *Proceedings of the Summer Program*, vol. 6, pp. 347–358. Center for Turbulence Research, Stanford University, 2010.

[3] V. Charton, A. Awad, J. Labaune. *Optimisation of a hybrid NS–DSMC methodology for continuous–rarefied jet flows.* Acta Astronautica, **195**, 295–308, 2022.

[4] R. O. Fox, F. Laurent. *Hyperbolic quadrature method of moments for the one-dimensional kinetic equation.* SIAM Journal on Applied Mathematics, **82**, 750–771, 2022.

[5] H. Grad. *On the kinetic theory of rarefied gases.* Communications on Pure and Applied Mathematics, **2**, 331–407, 1949.

[6] Z. Hu, Z. Cai, Y. Wang. *Numerical simulation of microflows using hermite spectral methods.* SIAM Journal on Scientific Computing, **42**, B105–B134, 2020.

[7] F. La Torre, S. Kenjereš, J.-L. Moerel, C. Kleijn. *Hybrid simulations of rarefied supersonic gas flows in micro-nozzles.* Computers & Fluids, **49**, 312–322, 2011.

[8] C. D. Levermore. *Moment closure hierarchies for kinetic theories.* Journal of Statistical Physics, **83**, 1021–1065, 1996.

[9] T. Schwartzentruber, I. Boyd. *A hybrid particle-continuum method applied to shock waves.* Journal of Computational Physics, **215**, 402–416, 2006.

[10] K. Xu, J.-C. Huang. *A unified gas-kinetic scheme for continuum and rarefied flows.* Journal of Computational Physics, **229(20)**, 7747–7764, 2010.

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