Backward Error Analysis of Isospectral Integrators via Lie–Poisson Reduction of Butcher Series

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Numerical simulations play a central role in understanding complex dynamical systems, from fluid motion to large-scale physical models. In many such systems, important quantities such as energy or spectral properties are preserved over time, and capturing these features accurately is essential for reliable long-term simulations.

In this talk, we present a new framework for analyzing a class of structure-preserving numerical methods known as isospectral symplectic Runge–Kutta methods. Our approach builds on classical backward error analysis and is formulated in terms of Butcher series, a powerful algebraic tool for describing numerical integrators. By introducing Lie–Poisson reduction of Butcher series, we connect these expansions to the geometry of Lie–Poisson systems, which naturally arise in many models from mechanics and fluid dynamics.

This perspective provides an intuitive explanation of how numerical methods inherit geometric structure from the underlying equations. In particular, it clarifies why key invariants are well preserved over long time intervals, by interpreting the numerical method as exactly solving a nearby system that retains the same Lie–Poisson structure.

Beyond its practical implications, the framework highlights deep connections between numerical analysis, geometry, and algebra, and offers new tools for the systematic study of structure-preserving algorithms. We will illustrate these ideas with examples motivated by fluid dynamics, emphasizing how geometric insight can inform the design of robust numerical methods.

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