Acceleration of Newton-like methods for nonlinear algebraic systems by preflattening techniques

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In this work, we endeavor to enhance the efficiency of Newton's method in solving large nonlinear algebraic systems arising from the numerical discretization of physical models. Over the last two decades, nonlinear *preconditioning* techniques have been introduced to speed up the resolution of nonlinear systems. They rely on solving an equivalent system that is judiciously built by analogy with the linear case. Nevertheless, unlike the linear case, there is no guarantee that the new system will be more suitable for Newtonian solution than the original one, even though this is generally observed in numerical experiments. In the linear case, it is the *condition number* of the matrix that governs the rate of convergence of the linear solver, and we can be sure that the new condition number will be more favorable. In the case of Newton's nonlinear iterations, this is no longer the case. We do not know exactly which scalar quantity has decreased between the old system and the new one. So, even if nonlinear preconditioning works, its foundation remains intuitive and heuristic, which hinders further improvements.
The main idea of our new approach stems from an observation that when the system is linear, Newton's method converges in a single iteration (at least in exact arithmetic). Based on this observation, we propose to explore transformations of the original system into an equivalent system that is "less nonlinear" in a quantitative sense to be specified via the definition of a *flatness number*. The latter is naturally related the rate of convergence of Newton. To distinguish this approach, which acts only on the external nonlinear level, from classical nonlinear preconditioning, which acts simultaneously on both the linear and nonlinear levels, we introduce the term *preflattening*.

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