Full-field simulation of a multiscale elliptic equation using Quantic Tensor Trains representations

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Abstract

This work addresses the numerical approximation of elliptic differential equations in heterogeneous media, with a particular focus on high-resolution discretizations and large computational challenges [2]. Our main motivation is handling multiscale problems with oscillating coefficient. The proposed framework relies on tensor-based techniques, in particular Quantized Tensor Train (QTT) representations [4, 5], to efficiently encode high-dimensional structures arising from fine discretizations. The originality of our approach lies in the construction of a QTT-based methodology that allows an accurate control of the gradient of the solution, even for very fine discretization, without relaying on an explicit mesh. This approach provides compact representations of both the problem data and the solution, while remaining flexible with respect to complex and heterogeneous coefficient fields. In addition, a comparison with other solvers [1, 3] is provided to demonstrate the efficiency of our approach. The results suggest that tensor-based solvers, and QTT representations in particular, offer a promising direction for large-scale simulations where classical methods may face limitations in terms of memory and computational cost.

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