The spectral domain decomposition method GenEO as a robust preconditioner for single-phase flow in fractured porous media

Pierre JOLIVET, LIP6, CNRS UMR 7606, Sorbonne Université - Paris, France
Michel KERN, Inria - Paris, France
Géraldine PICHOT, Inria - Paris, France
Frédéric NATAF, LJLL, CNRS UMR 7598, Sorbonne Université - Paris, France
Raphaël ZANELLA, Inria - Paris, France
Daniel ZEGARRA VASQUEZ, Université Paris-Saclay, CEA, Service d'Études Mécaniques et Thermiques - Gif-sur-Yvette, France

The Discrete Fracture Matrix (DFM) approach is a model for single-phase flow in fractured porous media in which fractures are explicitly represented as planar surfaces. The equations of the problem are the mass conservation and Darcy's law in the 3D rock matrix and in the 2D fracture network, coupled by a term modeling the exchange between the rock matrix and the fractures, and continuity equations at the intersection between the fractures. Discretizing this problem with the lowest order mixed-hybrid finite element method yields a sparse and symmetric positive definite linear system [4].

Most flow simulations in fractured porous media in the literature involve at most a few thousand fractures. When the number of fractures reaches tens of thousands, the linear systems become very large and are also highly ill-conditioned due to both the presence of poor quality elements in the mesh and, even more so, when there are strong conductivity contrasts between the rock matrix and the fractures. Direct solvers are then no longer suitable. Iterative solvers, preconditioned by algebraic multigrid methods and one-level domain decomposition (DD) methods such as Additive Schwarz (ASM) or Restricted Additive Schwarz (RAS), may take days to converge, or even stall under strong hydraulic conductivity contrasts [5].

Solving such linear systems efficiently, when they arise from models with a very large number of fractures, therefore requires a more sophisticated preconditioner. In this talk, we focus on the two-level DD method GenEO (Generalized Eigenvalue problem on the Overlap) [1], equipped with a spectral coarse space and combined at the fine level with either ASM or RAS. To apply GenEO to DFM flow problems, we propose a decomposition strategy tailored to the mixed-dimensional geometry of DFMs, which enables the construction of the Neumann matrices and mappings required by the method. The resulting GenEO-based preconditioner, with its HPDDM implementation in PETSc [3], consistently outperforms the methods mentioned above in both iteration count and wall-clock time [2]. In the largest case, involving 697k fractures, a maximum heterogeneity ratio close to 10^7 and 243M degrees of freedom, GMRES preconditioned with GenEO converges in 51 iterations in less than four minutes on 6825 MPI processes.

As part of the "France 2030" initiative, this work has benefited from a national grant managed by the French National Research Agency (Agence Nationale de la Recherche, ANR) attributed to the Exa-MA project of the NumPEx PEPR program, under the reference ANR-22-EXNU-0002. This work was granted access to the GENCI-sponsored HPC resources of TGCC@CEA under allocation AD010607519R2. This work was supported by the ANR through the project STEERS (Grant N° ANR-24-CE56-0127).

[1] P. Jolivet, F. Hecht, F. Nataf, C. Prud'homme. *Scalable domain decomposition preconditioners for heterogeneous elliptic problems.* In *Proceedings of the International Conference on High Performance Computing, Networking, Storage and Analysis*, SC '13. Association for Computing Machinery, New York, NY, USA, 2013. doi :10.1145/2503210.2503212.

[2] P. Jolivet, M. Kern, F. Nataf, G. Pichot, D. Zegarra Vasquez. *Domain decomposition preconditioners for efficient parallel simulations of single-phase flow in three-dimensional fractured porous media with a very large number of fractures.* Preprint hal-05029676, 2025.

[3] P. Jolivet, J. E. Roman, S. Zampini. *KSPHPDDM and PCHPDDM : Extending PETSc with advanced Krylov methods and robust multilevel overlapping Schwarz preconditioners.* Computers and Mathematics with Applications, **84**, 277–295, 2021. doi :10.1016/j.camwa.2021.01.003.

[4] M. Kern, G. Pichot, D. Zegarra Vasquez. *Mathematical and numerical analysis of the mixed formulation of single phase flow in three-dimensional fractured porous media.* Preprint hal-05029638, 2025.

[5] M. Kern, G. Pichot, D. Zegarra Vasquez. *Performance of algebraic preconditioners for large-scale simulations of single-phase flow in three-dimensional fractured porous media.* Preprint hal-05029652, 2025.

Contact : daniel.zegarravasquez@cea.fr