Modified equations for the long-time study of stochastic numerical schemes related to optimization

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In this talk, we consider a class of stochastic gradient descent schemes associated with an objective function F assumed to be strongly convex. We establish uniform-in-time weak error estimates between the numerical scheme and $x^\star$, the unique minimizer of F. To obtain optimal estimates, we introduce modified equations adapted to this setting, whose construction differs from existing approaches [1] and constitutes a novel contribution. These equations prove to be essential in achieving the optimality of the uniform-in-time bounds. This approach has also been developed for other stochastic numerical schemes in finite dimension, as well as for a scheme in infinite dimension. We illustrate the relevance of our results through various numerical simulations.

[1] E. Hairer, C. Lubich, G. Wanner. *Geometric numerical integration*, vol. 31 of *Springer Series in Computational Mathematics*. Springer-Verlag, Berlin, 2002. doi :10.1007/978-3-662-05018-7. Structure-preserving algorithms for ordinary differential equations.

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